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# WORKING PAPERS

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ACCOUNTABILITY,  
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## Abstract

*Formal electoral institutions occupy an important explanatory position in contemporary political science. Most such research examines the distinction between majoritarian and proportional electoral formulae. Recent work has also examined the impact of a system's ballot structure, i.e. the formal rules governing how citizens vote, on a variety of political phenomena. This paper develops a game theoretic model to study the interactive impact of electoral formulae and ballot structures on legislators' work habits. Among other things, the theoretical results help to resolve an ongoing debate as to the relative merits of First-Past-The-Post systems and Open-List voting systems in both constraining political corruption and generating personal legislative accountability. In contrast to received wisdom, they also identify a set of conditions under which Closed-List voting systems might themselves generate higher levels of personal legislative accountability than First-Past-The-Post systems. Analysis of cross-national data on political corruption provides support for the paper's theoretical model.\**

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## INTRODUCTION

Legislative electoral institutions occupy an important explanatory position in contemporary political science. Most studies of formal electoral rules examine the distinction between majoritarian (MAJ) and proportional (PR) *electoral formulae*.<sup>1</sup> The most common MAJ formula is the well-known First-Past-The-Post (FPTP) system used, among elsewhere, in Great Britain, Canada, and the United States. Recent research has also examined a system's *ballot structure*, i.e. the formal rules governing how citizens cast their votes. Of particular interest have been: a.) *preferential* rank-ordering systems which allow voters to identify not only their most-preferred candidate but also their second-most-preferred, third-most preferred, etc.;<sup>2</sup> and b.) *open-list* systems in which voters may simultaneously express support for political party organizations *and* particular candidates within these organizations' electoral lists.

While MAJ, PR, and preferential voting systems have been the subject of fairly extensive game theoretic analysis, much less work has aimed to model the combination of intra-party and inter-party competition characteristic of open-list electoral systems (Gingerich 2009 is a notable exception). Even less effort has been devoted to studying the MAJ/PR

distinction and open-list voting in *the same* theoretical model. After presenting a series of motivating empirical results (Section II), Sections III through VI develop a game theoretic model of legislative behavior which incorporates these distinct institutional parameters. Its primary theoretical hypotheses pertain to the relationship between *electoral institutions*, *political corruption*, and *personal legislative accountability*, the latter being defined as the extent to which legislators pursue the interests of geographic constituents rather than their own material or partisan agendas.

In contrast to recent arguments otherwise (see below), the model suggests that open-list systems' unique combination of intra- and inter-party competition should generate lower levels of corruption and higher levels of legislative accountability than *both* FPTP systems *and* 'closed-list' electoral systems. Also counter to received wisdom, the model identifies conditions under which 'closed-list' systems might themselves actually outperform FPTP systems in generating constituency-level accountability, highlighting both the size of multi-member districts and political parties' internal nomination procedures as crucial intervening variables. Section VII concludes.

Past literature on the consequences of electoral institutions is vast. One stream of research investigates the effect of electoral institutions on a country's *party system*. Early work by Duverger (1952) on the relationship between electoral formulae and party-system fragmentation has since been formalized and qualified by Riker (1982), Palfrey (1989), and Cox (1994, 1997).<sup>3</sup> In a distinct set of papers Cox (1987, 1990) investigates the consequences of electoral formulae and preferential rank-scoring

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<sup>1</sup> In MAJ systems Legislators are elected via some form of plurality counting rule such that the candidate(s) with the most votes earns political incumbency. In PR systems, used in a majority of continental European states, political parties are allocated legislative seats in multi-member districts according to their percentage of the district-level vote.

<sup>2</sup> In this category one finds both rank-scoring rules, such as the Borda Count and the Negative Vote; and vote-transferring rules such as the Alternative Vote (used in Australia) and the Single-Transferable Vote (used in Ireland and Malta). Under the latter a voter's support for his or her most-preferred candidate is transferred to another candidate once this most-preferred option's fate is decided. Under the former candidates are assigned 'points' in accordance with how many first-preference votes they receive, how many second-preference votes they receive, etc; but support never transfers from one candidate to another.

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<sup>3</sup> Duverger famously argued that single-member district FPTP systems should generate two-party competition (Duverger's Law), while PR systems with multi-member districts should facilitate the advent of party systems with more than two viable organizations (Duverger's Hypothesis). Cox (1997) formalizes the latter: in a district which sends  $M$  legislators to the national Legislature, he establishes that no more than  $\{M+1\}$  viable political parties should compete.

systems for party system polarization.<sup>4</sup> In turn, both the number of parties and their systemic polarization comprise important mechanisms in studies of PR's consequences for *ethnic conflict*.<sup>5</sup>

Another body of work investigates the consequences of electoral institutions for *economic policy*. A series of articles addresses the relative merits of PR as opposed to MAJ systems in generating socio-economic redistribution (Austen-Smith 2000; Soskice and Iversen 2005; Long-Jusko 2009). As well, Persson and Tabellini (2000; 2003) argue that PR competition favors the production of 'public good' policies applicable to society as a whole, whereas MAJ competition favors the production of more decentralized 'club good' policies targeted to individual geographic constituencies.<sup>6</sup> Finally, Myerson (1993a) argues that pure plurality rule should generate economic policies more narrowly targeted to exclusive social

minorities than those in preferential systems such as the Alternative Vote, the Borda Count, and the Negative Plurality Vote.

As is clear from the preceding paragraphs and footnotes, most work on the consequences of electoral institutions for party systems, ethnic conflict, and economic policy has emphasized the causal effects of electoral formulae and preferential rank ordering systems, to the exclusion of open-list systems which permit intra-party candidate voting. While early work on the relationship between electoral rules and *political corruption* similarly emphasized the impact of MAJ vs. PR electoral formulae (Myerson 1993b; Lijphart 1999; Persson and Tabellini 2000), more recent contributions have distinguished between open-list PR systems (OLPR) in which voters may simultaneously express support for a political party and a particular candidate within that party's electoral list; and closed-list PR systems (CLPR) in which voters may not express preferences for specific candidates within a party's list (Persson *et al.* 2003; Persson and Tabellini 2003; Kunicova and Rose-Ackerman 2005).

A growing consensus in this literature claims that "...proportional representation (PR) systems are more susceptible to corrupt political rent-seeking than plurality systems."<sup>7</sup> More particularly, the claim is that FPTP elections generate stronger ties of personal accountability between legislators and constituents than *both* OLPR and CLPR systems, which in turn makes FPTP particularly effective at constraining political corruption. These same papers argue that OLPR produces levels of legislative accountability and political corruption *intermediate* to high accountability FPTP systems and low accountability CLPR systems. Consider the following quotations:

- a) "The possibility of holding individual politicians accountable through open-lists seems a less powerful deterrent [for corruption] than individual ballots associated with plurality rule." (Persson and Tabellini 2003; pgs. 195-196)

<sup>4</sup> Among other results he establishes that multi-candidate plurality rule elections generate non-centrist policy platforms; identifies a set of preferential rank-scoring rules that generate centrist equilibria in multi-candidate contests; and argues that PR systems with large district magnitudes should generate fairly polarized party competition.

<sup>5</sup> Beginning with Lijphart (1977) proponents of *consociationalism* have argued that multi-party competition in PR systems facilitates minority representation, which in turn allows for the functional mediation of potentially conflictual ethnic cleavages. In contrast, Horowitz (1985), Reilly (2001), and others have touted the mediating capacity of preferential vote-transferring systems such as the Alternative Vote employed in Australia and the Single-Transferable Vote employed in Ireland. Fraenkel and Grofman (2004) develop a game theoretic argument as to the conditions under which alternative voting does, and does not, yield outcomes compatible with moderation and ethnic accord.

<sup>6</sup> See Milesi-Ferretti *et al.* (2002) for a similar argument (albeit conceptually reversed: they refer to geographically targeted policies as 'public goods'). In contrast, Rogowski and Kayser (2002) argue that fiscal policy in PR systems should tend to target resource-rich but vote-poor special interest groups, while policy in MAJ systems should represent the preferences of socially diffuse but vote-rich consumers.

<sup>7</sup> Kunicova and Rosa-Ackerman *ibid.*, pg. 573.

- b) “Because open-list proportional representation systems share features of both closed-list proportional representation and plurality systems, they occupy an ‘intermediate’ category in monitoring corrupt self-enrichment.” (Kunicova and Rose-Ackerman *ibid*; pg. 585)

Interestingly, the notion that FPTP systems generate higher levels of personal legislative accountability than OLPR systems contradicts well-known arguments in the comparative study of legislative behavior, which posit that OLPR should generate substantially *higher* levels of personal legislative accountability than many common FPTP systems, including those found in the UK and Canada (Carey and Shugart 1995).<sup>8</sup> The following Sections present both empirical and theoretical evidence that, in contrast to the growing consensus noted above, OLPR systems generate both higher levels of legislative accountability and lower levels of political corruption than their FPTP and CLPR counterparts.

### **ELECTORAL FORMULA, BALLOT STRUCTURE, AND POLITICAL CORRUPTION**

The primary dependent variable employed in the above-quoted studies of electoral rules and political corruption is the *Control of Corruption Index*, which itself is one of six *Governance Indicators* compiled by World Bank researchers over the past 15 years (Kauffman *et al.* 2008).<sup>9</sup> Evidence in

all of the above-cited studies comes from cross-national investigation of data from 1997-1998, facilitating both statistical replication and the parsimonious evaluation of competing hypotheses. Persson and Tabellini (2003) and Persson *et al.* (2003) contain largely identical empirical analyses; henceforth I confine myself to the former so as to avoid redundancy. The authors use three institutional measures to study the relationship between a country’s electoral formula, its ballot structure, and political corruption: MAJ, PIND, and PINDO. MAJ captures whether or not a country uses some form of *plurality rule* to elect its legislators, such that [MAJ=1] in plurality rule systems while [MAJ=0] in PR systems.<sup>10</sup> PIND captures the percentage of a country’s legislators who are elected as individual candidates *independent of party lists*, such that [PIND=1] in FPTP with single-member districts systems [PIND=0] in pure PR systems and other party list systems (PIND may assume values between 0 and 1 in countries which use a mix of party lists and direct candidate voting).<sup>11</sup> The third variable employed, PINDO, is a variant of PIND which accounts for the fact that legislators in OLPR systems occupy party lists, but are also the recipients of individually targeted candidate votes. It captures the percentage of legislators in a particular country which *are not* elected using *closed-party lists*, such that [PINDO=1] for both FPTP and OLPR

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subjective corruption measures, my goal in this Section is not to provide a definitive account of political corruption’s empirical correlates, but rather: a.) to demonstrate that the aforementioned conventional wisdom grounded in analyses of these subjective measures is subject to criticism; and b.) to provide suggestive evidence that the actual institutional correlates of political corruption match more closely the theoretical predictions derived in Sections III through VI.

<sup>10</sup> Far and away the most common form of plurality rule is FPTP in single-member districts (see below).

<sup>11</sup> As the authors readily admit, the variables MAJ and PIND are highly, though not perfectly, correlated ( $r = .926$ ) due to the fact that by far the most common plurality rule system is FPTP in single-member districts, where legislators are *by definition* not elected on party lists.

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<sup>8</sup> In this vein, one of the few existing formal models of OLPR (Gingerich *ibid*) notes its role in empowering district-level voters and reducing legislative candidates’ incentives to engage in corrupt behavior.

<sup>9</sup> This oft-used index aggregates into a single measure information from over 30 distinct public opinion and professional surveys which ask respondents for their subjective evaluations of a particular country’s experience with political corruption. Triesman (2007) contains a detailed account of the strengths and weaknesses of this and other data sources on political corruption. Though cognizant of the potential pitfalls associated with these

systems, while [PINDO=0] for CLPR systems.

I begin by replicating the results found in Table 7.1 on pages 192-193 of Persson

and Tabellini (2003). The dependent variable is GRAFT, a transformed measure of the World Bank's corruption index for which higher values indicate a greater

**TABLE 1. Weighted-Least-Squares Analysis of GRAFT (N=78)**

PIND	- 1.783 ** (.894)		.601 (.633)	.096 (.655)	- .442 (.419)	- .719 (.467)
PINDO		- .385 (.279)				
MAJ	- .563 (.592)	- 1.208 ** (.456)	- .857 (.639)	- .473 (.643)		
MAGN	2.762 *** (.787)	1.679 *** (.526)				
OLPR				- .597 ** (.271)	- .638 ** (.264)	- 1.208 ** (.506)
FPTP					.140 (.412)	- .153 (.466)
CLPR						- .656 (.498)
PRES	- .699 ** (.278)	- .608 ** (.284)	- .508 * (.298)	- .408 (.292)	- .398 (.294)	- .397 (.292)
FEDERAL	.185 (.302)	.179 (.308)	.189 (.330)	.278 (.322)	.340 (.315)	.252 (.321)
GASTIL	.076 (.164)	.083 (.167)	.107 (.179)	.038 (.176)	.028 (.177)	.059 (.178)
AGE	- .210 (.628)	- .029 (.629)	- .008 (.682)	- .183 (.666)	- .292 (.664)	- .277 (.660)
COL_UK	- .776 *** (.285)	- .591 ** (.283)	- .472 (.296)	- .567 * (.290)	- .631 ** (.279)	- .701 ** (.282)
PROT80	- .010 ** (.005)	- .009 * (.005)	- .007 (.005)	- .007 (.005)	- .007 (.005)	- .007 (.005)
CATHO80	.003 (.004)	.004 (.004)	.004 (.004)	.003 (.004)	.003 (.004)	.003 (.004)
CONFU	1.417 *** (.522)	.994 ** (.463)	.559 (.504)	.662 (.491)	.795 (.478)	.693 (.482)
AVELF	1.012 * (.564)	.661 (.559)	.419 (.587)	.383 (.569)	.378 (.573)	.450 (.572)
LPOP	.026 (.114)	.014 (.116)	.043 (.135)	.036 (.121)	.023 (.122)	.030 (.121)
EDUGER	- .012 (.008)	- .011 (.008)	- .001 (.009)	- .011 (.008)	- .011 (.008)	- .012 (.008)
LYP	- .859 *** (.234)	- .958 *** (.231)	- 1.026 *** (.250)	- 1.016 *** (.290)	- .984 *** (.242)	- .930 *** (.244)
TRADE	- .004 (.003)	- .004 (.003)	- .004 (.003)	- .004 (.003)	- .005 (.003)	- .005 (.003)
OECD	- 1.155 *** (.418)	- 1.124 *** (.425)	- 1.039 ** (.455)	- 1.150 ** (.444)	- 1.186 *** (.442)	- 1.156 ** (.440)
LAAM	.976 ** (.396)	.765 * (.416)	.748 * (.426)	.597 (.419)	.555 (.416)	.680 (.424)
CONS	12.56 ***	13.44 ***	14.04 ***	14.44 ***	14.35 ***	14.47 ***

Standard errors in parentheses; (p>.10) = \* ; (p>.05) = \*\* ; (p>.01) = \*\*\*.



presence of corruption, with a mean of 4.14 and standard deviation of 1.89 (ranging from a low of .74 in Denmark to a max of 6.92 in Paraguay). The statistical model is *weighted-least-squares*, where all regressions are weighted by the *inverse standard deviation* of the surveys which enter into the original index, to control for the fact that some countries generate higher levels of subjective uncertainty than others. The measurement specifics for GRAFT, MAJ, PIND, PINDO, and all controls can be found in Appendix A.

Columns 1 and 2 replicate the findings which motivate the authors' most basic empirical conclusions.<sup>12</sup> The results in column 1 come from a regression which includes both MAJ and PIND. Both coefficients are negative, which conforms to the authors' theoretical expectations: the direct legislative accountability associated with plurality rule should *reduce* corruption, while the muted accountability associated with party list competition should *increase* corruption. However only PIND attains statistical significance, due probably to the two measures' multi-collinearity ( $r = .926$ ; see fn 11). Column 2 presents results from a regression with replaces PIND with PINDO, whose correlation with MAJ is lower ( $r = .680$ ); once again both coefficients are negative, but in this case only MAJ attains statistical significance, i.e. embedding open-list considerations into the variable PIND dilutes its statistical effect, and makes MAJ the most robust predictor. These results motivate the conclusion, quoted above, that plurality rule systems without party lists outperform *both* OLPR and CLPR in constraining corruption.

The first thing to note about these regressions is that they both contain the variable MAGN, which captures a

country's *inverse district magnitude*, i.e. its number of electoral districts divided by its total number of legislative seats, such that [MAGN=1] in pure single-member district systems and [MAGN<1] in systems with at least one multi-member district. Not surprisingly, this variable is itself highly correlated with both MAJ ( $r = .886$ ) and PIND ( $r = .928$ ). Thus, the regression in column 1 contains three institutional measures correlated with one another at roughly  $r = .9$ , which makes the substantive interpretation of statistical coefficients a challenge. Column 3 contains the results of a regression identical to column 1 save for the exclusion of MAGN; without the inclusion of this highly multicollinear variable neither PIND nor MAJ attains statistical significance, and the sign on the former becomes positive.<sup>13</sup>

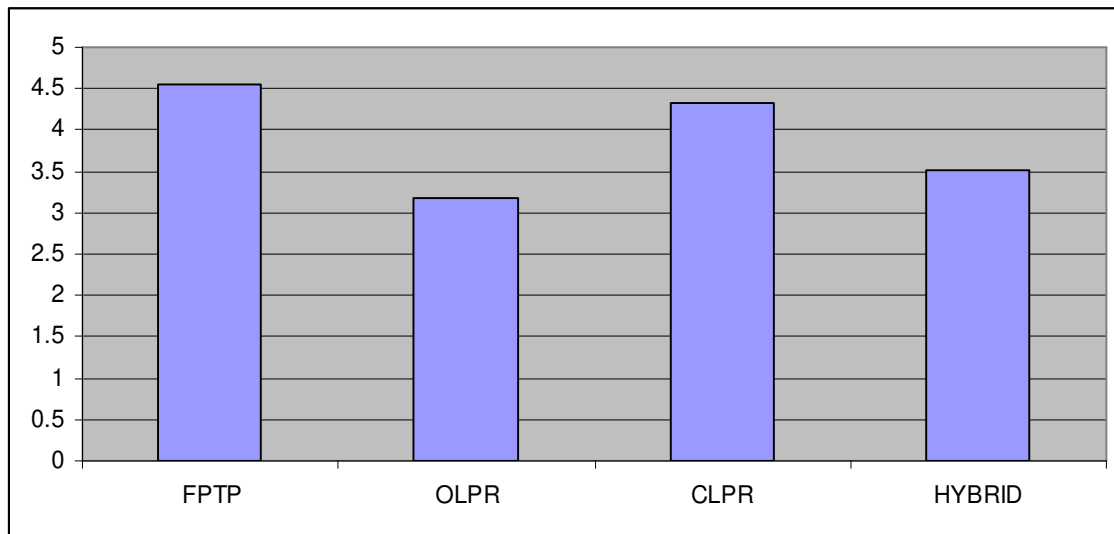
The second noteworthy aspect of this analysis is that the measures employed to capture a country's electoral formula and ballot structure group together systems with very different strategic properties. For example, despite the fact that the authors' cited theoretical models apply only to FPTP systems, the variable MAJ groups together FPTP systems with the Alternative Vote used in Australia and the Bloc Vote used in Mauritius and Thailand.<sup>14</sup> As well, the variable PIND regroups countries which

<sup>13</sup> The nearly non-existent correlations between GRAFT and both PIND ( $r = .038$ ) and MAJ ( $r = .059$ ) lend support to the suspicion that these variables' statistical significance in columns 1 and 2 is largely a result of the institutional measures' multi-collinearity.

<sup>14</sup> As well, Chile is coded as a plurality rule system despite the fact that it has, since 1988, used the equivalent of OLPR in two-member districts (with d'Hondt as the effective formula) to elect its legislatures. Similarly, South Korea is coded as *non-plurality-rule* despite the fact that it has been a predominantly FPTP system since its transition to democracy. South Korea elects 245 legislators in single-member FPTP districts, and another 54 in a national-level upper-tier. However, the seat allocations in this upper-tier are based on one's success in the FPTP districts (i.e. the more votes one gets in FPTP districts the more seats one receives in the upper-tier), such that parties' electoral calculations are driven nearly completely by single-member-district calculations.

<sup>12</sup> Despite repeated attempts using all possible combinations of relevant control variables, I was unable to generate the exact weighted-least squares coefficients presented 7.1 of Persson and Tabellini (2003). As such, I settled on the set of control variables used by the authors themselves. The results presented here are nearly identical in terms of both substantive size and statistical significance to the original results.

**FIGURE 1. Mean Comparison of Graft**



use CLPR, OLPR, and a variety of hybrid list systems including aforementioned Bloc Vote and the Single-Transferable-Vote used in Ireland and Malta. Finally, the variable PINDO regroups FPTP systems and OLPR systems. The theoretical results derived in Sections III-VI demonstrate the potential hazards of this systemic conflation. I have thus created the variables FPTP, OLPR, CLPR, and HYBRID, each of which measures the percentage of a country's legislators elected under the relevant system (HYBRID groups together systems such as the Alternative Vote, Single-Transferable Vote, and Bloc Vote).<sup>15</sup> Appendix A presents all countries' individual values on these variables, along with coding rules used in their creation. In most cases a country's value on these variables is either 0 or 1; the few countries which used mixed systems have fractional values on two of these measures. For the latter cases, define a country's *predominant system* as the system used to elect a majority of its legislators.<sup>16</sup> In the 84 country dataset, 30

countries are completely or predominantly FPTP, 33 are CLPR, 14 are OLPR, and 6 are HYBRID. As a starting point consider FIGURE 1, which presents the mean values of GRAFT among all countries of a particular predominant system.

This simple mean comparison defies quite strikingly the aforementioned conventional wisdom: mean levels of corruption are much lower in OLPR systems than in any other system, and are lower than those found in FPTP systems by nearly  $\frac{3}{4}$  of a standard deviation on the GRAFT scale (3.17 as opposed to 4.56). In fact, FPTP systems, lauded in previous studies, register a larger mean level of corruption than *any* category.

Of course, simple mean comparisons are often misleading. Columns 4, 5, and 6 from Table 1 introduce the interval measures OLPR, FPTP, and CLPR into a more rigorous statistical setting. As my goal is a comparative evaluation of hypotheses pertaining to electoral formulae and ballot structure, I exclude MAGN in these regressions to mitigate problems of multicollinearity.<sup>17</sup> When introduced into a

<sup>15</sup> In keeping with the dependant variable's time point (1997-1998), these variables represent the system used in a particular country during the years 1994-1997.

<sup>16</sup> In almost all cases the coding of a country's predominant system is straight-forward. For example, Poland and Switzerland register scores of .85 (.15) and .975 (.025) respectively on the variable OLPR (FPTP), such that the predominant rule is clearly OLPR although both

countries contain a small number of single-member-districts. Russia is the unique case in which the system is perfectly divided (both CLPR=.5 and FPTP=.5), and is thus not included in Figure 1's mean tally.

<sup>17</sup> I have run all regressions with both MAGN and PINDO included. PINDO has no effect on any of the following results. The inclusion of

weighted-least-squares regression along with PIND and MAJ (column 4), only OLPR has a significant and reductive effect on a country's overall level of political corruption. Column 5 replaces the variable MAJ with the variable FPTP, which more precisely operationalizes the theoretical arguments referred to in past studies. Once again, neither FPTP nor PIND has a statistically significant effect on GRAFT, while OLPR has a significant reductive effect. Finally, column 6 introduces CLPR into the mix, with a similar qualitative result: only OLPR has a significant reductive effect on corruption. In this last column, this effect becomes substantively stronger, such that moving from a system with [OLPR=0] to a system with [OLPR=1] reduces corruption by two-thirds of a standard-deviation in the GRAFT measure.<sup>18</sup> To summarize, these empirical findings cast much doubt on *both* the notion

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MAGN does not reduce the statistical or substantive significance of OLPR; but does return the highly multicollinear PIND to its previous statistical significance (see above).

<sup>18</sup> These results also differ substantially from those uncovered in Kunicova and Rose-Ackerman (ibid). Although for reasons of space I do not conduct a full replication of their analysis, a number of things can be said about these differences. Firstly, the authors misclassify a number of cases, coding for example Chile as a plurality-rule system and Poland as a CLPR system (both in fact use OLPR), and coding Hungary and Guatemala as plurality-rule systems despite the fact that both are predominantly CLPR systems. More generally, the authors conflate pure FPTP systems with HYBRID systems such as the Alternative Vote and the Bloc Vote; and in all cases the authors use dummy variables which fail to capture the mix of rules used in mixed electoral systems. However, perhaps the most important difference between their analysis and that presented here is the sample size: their sample contains a non-negligible number of countries which are excluded from Persson and Tabellini's analysis due to their lack of democratic credentials (e.g. Jordan, Kazakhstan, Kuwait, Sierra Leone, Zimbabwe, Yemen, etc.). Thus, beyond the measurement issues noted above, behind our contradictory results lies an unresolved question, which exceeds my current scope, as to the consequences of electoral institutions in semi-democracies and/or non-democracies.

that FPTP systems generate lower levels corruption than PR systems, *and* the notion that OLPR occupies an intermediate position between high accountability FPTP systems and low accountability CLPR systems. In contrast, OLPR outperforms both FPTP and CLPR, while the latter two systems are statistically indistinguishable. The following game theoretic model helps to explain why OLPR might outperform its counterparts as such, and also generates a series of hypotheses as to the *contingent* consequences of FPTP and CLPR.

## LEGISLATORS, VOTERS, AND ELECTIONS IN FIRST-PAST-THE-POST SYSTEMS

Consider a country composed of  $N$  evenly-populated geographic *regions*, which are then aggregated into some number  $D$  electoral *districts*. I will use the marker  $d \in \{1, 2, \dots, D\}$  to identify electoral districts, and the marker  $j \in \{1, 2, \dots, M_d\}$  to denote individual regions within particular electoral districts, where  $M_d$  is defined as the number of regions contained inside district  $d$ 's boundaries. In FPTP systems with single-member constituencies, each geographic region will comprise an individual electoral district, such that  $D = N$  and  $M_d = 1$ . Countries such as Israel and the Netherlands have electoral systems with a single national electoral district, such that  $D = 1$  and  $M_d = N$ .

The model's primary strategic actors will be *incumbent legislators*, who must allocate scarce resources in the pursuit of distinct, and often mutually exclusive, political and material goals (Fenno 1978). This paper emphasizes two such behavioral options:

- a) devoting effort to the provision of particularistic goods and services for district constituents (e.g. pork projects, social services, ombudsman services, etc);
- b) and devoting effort to the pursuit of one's own personal enrichment (e.g. through bribery, embezzlement, and other forms of political corruption).

Section VII then extends the argument to situations in which legislators have a third behavioral alternative: devoting effort to the development and implementation of a party's national-level public policies. Section VII also addresses the frequent public and scholarly conflation of (a) and (b), conflation grounded in the notion that legislative particularism and political corruption are causally related and/or conceptually indistinct.

Before proceeding, it is important to spend a moment addressing the notion of 'constituency service' put forth in item (a). In FPTP systems, individual incumbents are clearly affiliated with individual electoral districts, and their particularistic efforts on behalf of constituents will, by definition, be targeted to voters in their particular district. On the other hand, in *multi-member district* systems, it is generally impossible for incumbent legislators to develop particularistic relationships with *all* voters in these substantially larger electoral districts. For example, research on countries as varied as Ireland (Martin 2010), Brazil (Ames 1995), Argentina (Swarczberg 2009), Columbia (Ingall and Crisp 2001), and Turkey (Kselman 2009) suggests that individual incumbents from multi-member districts target their particularistic efforts to well-defined regional or municipal strongholds (*aka* 'bailiwicks') within larger electoral districts.<sup>19</sup> To model the process by which legislators in multi-member districts develop particularistic relationships, I will thus make the following assumption: individual incumbents each have *exactly one* region within their larger electoral district which they *may* target with particularistic goods and services (as demonstrated below, in equilibrium incumbents often devote *no* effort to legislative particularism).<sup>20</sup>

<sup>19</sup> It may also be the case that incumbents from multi-member districts target particularistic efforts to members of a well-defined professional group (e.g. Scheiner 2007 on Japanese legislative politics).

<sup>20</sup> For systems in which targeting occurs on a professional (or ethnic...) rather than a geographic basis, one could re-interpret the  $j \in \{1, 2, \dots, M_d\}$  individual regions inside district  $d$ 's boundaries as individual

In order to operationalize this theoretical format, let the size of a country's national Legislature be equal to the number of its geographic regions (i.e. it contains  $N$  incumbent legislators); and let the indicator  $M_d$  represent not only the number of regions within an electoral district, but also that district's *magnitude*: the number of legislators which it sends to the Legislature. In FPTP systems, where  $D = N$  and  $M_d = 1$ , each incumbent will have the option of exerting particularistic effort on behalf of voters in a single-member electoral district. In multi-member-district systems, where  $D < N$  and  $M_d > 1$ , each incumbent will have the option of exerting particularistic effort on behalf of a single region within a larger electoral district.

For the sake of presentation, in the text I examine a game in which each of these  $N$  incumbent legislators is affiliated with one of *two* political parties  $P \in \{A, B\}$ . Having solved the two-party game in Theoretical Appendices B-E, Theoretical Appendix F demonstrates that the model's comparative static implications are *identical* when more than two political parties compete. For an incumbent affiliated with party  $P$ , district  $d$ , and region  $j$ , denote  $f_{j,d}^P$  as the level of effort devoted to securing regional voters' particularistic interests; and  $c_{j,d}^P$  as the level of effort devoted to securing one's own material enrichment. All legislative incumbents from party  $P$  are endowed with a fixed amount of effort  $E^P$  which they divide exhaustively between  $f_{j,d}^P$  and  $c_{j,d}^P$ , implying the binding effort constraint  $f_{j,d}^P + c_{j,d}^P = E^P$ .<sup>21</sup> Define  $\mathbf{F}_d = \{f_{1,d}, f_{2,d}, \dots, f_{M_d,d}\}$

professional groups, and allow incumbents each to target an individual professional category.

<sup>21</sup> By allowing effort capacities to vary across parties I am able to make explicit a variety of empirical regularities, the most obvious being that members of the legislative majority are likely to have greater access to legislative resources than those in the minority party. It is also possible to let effort capacities vary across individual incumbents without changing any of the paper's results.

as a *strategy vector* containing the constituency service allocations of all incumbents from a particular district  $d$  (I omit party superscripts in strategy vectors for notational ease), such that  $\mathbf{F}_d = f_{1,d}^P$  in FPTP systems.

Legislators will allocate this effort with an eye towards the pursuit of two principal objectives: the desire for *re-election* and the desire for personal *material wealth*.<sup>22</sup> For an incumbent affiliated with party  $P$ , and with region  $j$  inside district  $d$ , consider the following utility function which links legislative effort allocation decisions to the pursuit of both re-election and personal material wealth:

$$U_{j,d}^P = c_{j,d}^P + [\pi_{j,d}^P(\mathbf{F}_d) \cdot \beta_{j,d}^P], \quad (1)$$

where  $\pi_{j,d}^P(\cdot)$  represents the legislator's probability of gaining re-election and  $\beta_{j,d}^P$  represents the benefit he or she associates with re-election. Legislators' utility thus increases linearly with effort devoted to the acquisition of material wealth.<sup>23</sup> On the other hand, as formalized in the model of voter choice below, effort devoted to  $f_{j,d}^P$  provides indirect and contingent benefits by increasing legislators' support among citizens in their affiliated regions, which may in turn increase  $\pi_{j,d}^P(\cdot)$ . As demonstrated in a previous working paper (Kselman 2008a), an additional term incorporating incumbents' preferences for their *party's overall electoral performance* can be added to (1) without significantly changing the upcoming theoretical results. I adopt here the simpler version.

In the below game's first stage, all  $N$  legislators will simultaneously allocate their fixed amount of effort  $E^P$  between  $f_{j,d}^P$  and  $c_{j,d}^P$ . In the second stage an election is

held and citizens choose between parties  $A$  and  $B$ . Voter choice will be influenced by *both* their regional incumbent's first-stage effort allocation and their own 'partisan' biases for or against parties  $A$  and  $B$ , biases determined by political considerations *independent* of their regional incumbent's effort allocation.<sup>24</sup> I follow Persson and Tabellini (2000) in modeling such organizational preferences with a single parameter capturing voter  $i$  in region  $j$ 's *partisan attitude*. Label this partisan attitude  $\sigma_{i,j}$ , and let higher values of  $\sigma_{i,j}$  correspond to more favorable attitudes for party  $A$ , and lower values to more favorable attitudes for party  $B$ .

Sections III through VI treat these partisan attitudes as exogenous and ask the following question: *given* some distribution of partisan attitudes in the electorate, what are the equilibrium levels of  $f_{j,d}^P$  and  $c_{j,d}^P$  provided by parties' current legislative incumbents? Section VII then extends the argument to situations in which partisan attitudes emerge endogenously. Until then, assume for expository purposes that partisan preferences in region  $j$  vary according to a *uniform distribution* over the support set  $[\underline{\sigma}_j, \bar{\sigma}_j]$ ,<sup>25</sup> where  $\bar{\sigma}_j$  represents the attitude of the voter in region  $j$  who is most inclined to choose party  $A$ , and  $\underline{\sigma}_j$  that of the voter in region  $j$  most inclined to choose party  $B$ . As well, without loss of generality assume that  $-1 < \underline{\sigma}_j < 0 < \bar{\sigma}_j < 1$  and that  $\bar{\sigma}_j - \underline{\sigma}_j = 1$ , where the latter implies that both the 'height' and the 'width' of the uniform distribution in region  $j$  are equal to 1. For example, given these distributional assumptions, a region whose partisan

<sup>22</sup> Kselman (2008a) discusses extending the model to situations in which legislator's also have preferences for particular public policy positions.

<sup>23</sup> The model is robust to the equally common assumption that individuals' utility for wealth exhibits decreasing marginal returns.

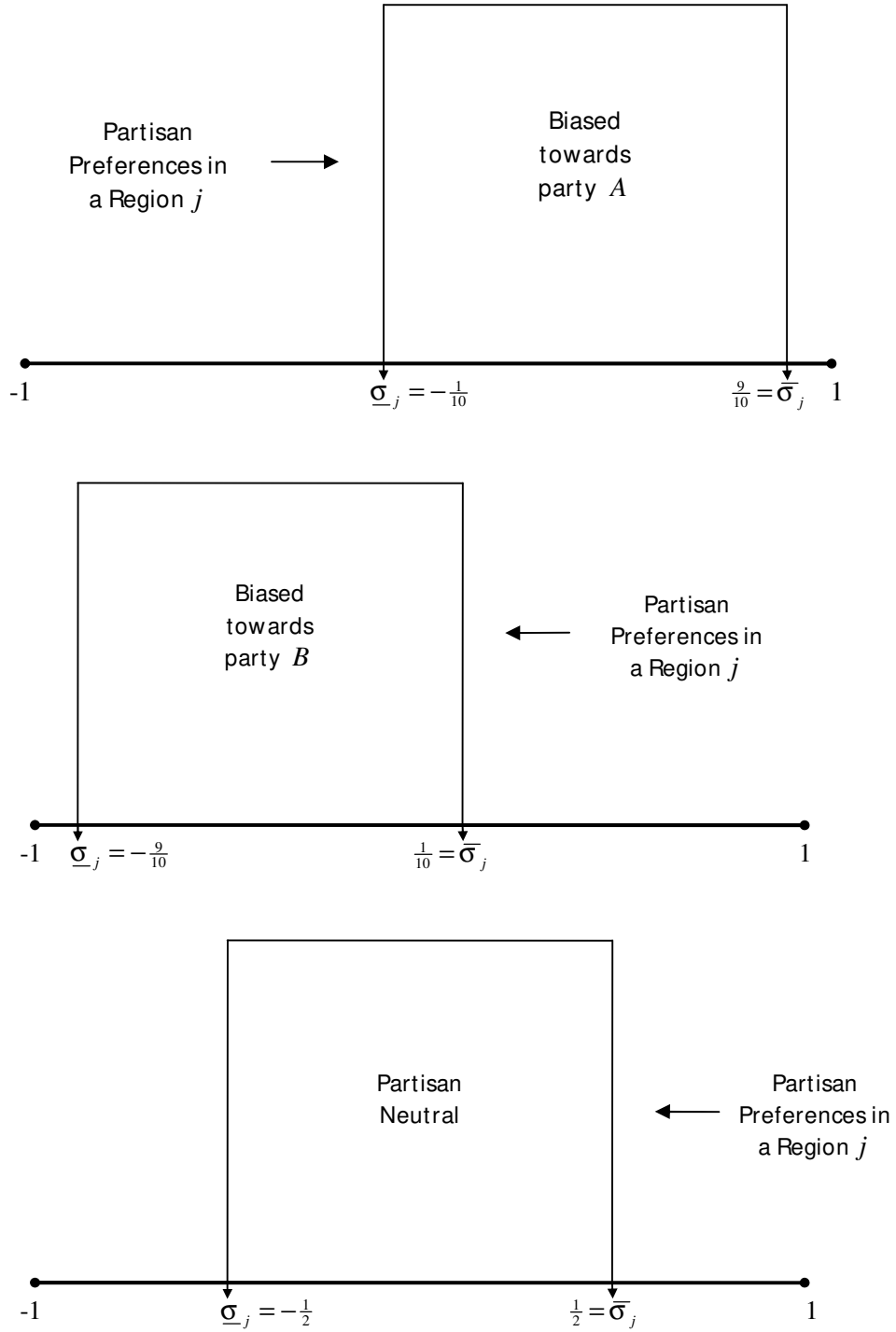
<sup>24</sup> Most obvious among such considerations are voter preferences for parties' respective national-level policy platforms. Also potentially relevant are voters' symbolic and affective 'identification' with one party or another, grounded for example in family history, the party's ideological/historical legacy, etc.

<sup>25</sup> Though the model is robust to alternative distributional assumptions, the straight-forward calculus of uniform distributions greatly simplifies the analysis.

attitudes are distributed over the support set  $[-\frac{1}{10}, \frac{9}{10}]$  is heavily 'biased' towards party  $A$ ; a region with partisan support set  $[-\frac{9}{10}, \frac{1}{10}]$  is heavily 'biased' towards party

$B$ ; and a region with partisan support set  $[-\frac{1}{2}, \frac{1}{2}]$  is 'neutral'.

**FIGURE 2. Regional Distributions of Partisan Attitudes**



I will model voter choice as the decision to ‘accept’ or ‘reject’ *the party of their regional incumbent*, i.e. the party of the legislator who may provide their region particularistic goods and services.<sup>26</sup> Given this approach, I must specify distinct utility functions for voters depending on the party affiliation of the relevant regional incumbent. Begin with a voter  $i$  in region  $j$  whose regional incumbent is affiliated with party  $A$ :

$$u_{i,j}^A = f_{j,d}^A + \sigma_{i,j}. \quad (2)$$

Voter utility thus increases linearly with both the level of goods/services targeted to his or her region and with his or her partisan attitudes. Recall that, by construction, higher values of  $\sigma_{i,j}$  correspond to more favorable attitudes for party  $A$ . For this reason  $\sigma_{i,j}$  enters (2) in an additive manner, such that the utility for party  $A$  is higher among voters with higher values of  $\sigma_{i,j}$  (and vice versa). A similar specification of voter utility is applicable in regions whose legislative incumbent is affiliated with party  $B$ :

$$u_{i,j}^B = f_{j,d}^B - \sigma_{i,j}. \quad (3)$$

This expression differs from (2) in that partisan attitudes enter as a subtracted rather than an additive term, since by construction lower values of  $\sigma_{i,j}$  correspond to more favorable attitudes for party  $B$ . Utility for party  $B$  is thus higher among voters with lower values of  $\sigma_{i,j}$  (and vice versa); and voters for whom  $\sigma_{i,j} < 0$  will have ‘positive’ utility for  $B$ .

Voters must thus ask themselves whether or not the combined satisfaction derived from an incumbent’s first-stage

constituency service effort  $f_{j,d}^P$  and their own partisan attitude  $\sigma_{i,j}$  is sufficient to vote for this incumbent’s party in the game’s second-stage election. The notion of a *reservation utility* provides a useful mechanism for modeling the process by which voters make this assessment. Define the reservation utility  $\eta$  as the satisfaction level at which voters feel sufficiently pleased with the party of their region’s incumbent legislator to choose that party in the game’s election. As such, in the game’s electoral stage, voters in regions whose incumbent is from party  $A$  ( $B$ ) will vote for this party if  $u_{i,j}^A \geq \eta$  ( $u_{i,j}^B \geq \eta$ ).

Without loss of generality I will normalize the game’s reservation utility to  $\eta = 0$ . In turn, for a region  $j$  whose incumbent is from party  $A$ , given some effort allocation  $f_{j,d}^A$  in the game’s first stage we can derive  $V_{j,d}^A(f_{j,d}^A)$ , party  $A$ ’s regional *vote percentage* in the game’s subsequent electoral stage (i.e. the portion of  $j$ ’s voters for whom  $u_{i,j}^A \geq 0$ ). Similarly, given  $f_{j,d}^B$  we can derive  $V_{j,d}^B(f_{j,d}^B)$ , party  $B$ ’s vote percentage in a region whose incumbent is from party  $B$ . These vote share expressions are presented in Lemma 1, whose formal derivation is contained in Appendix B.

<sup>26</sup> One might object that in PR systems voter choice between competing parties often has little to do with legislative particularism; as will become evident below, in many systems this very pattern emerges in equilibrium, i.e. voter choice is grounded completely in their partisan attitudes  $\sigma_{i,j}$ .

**LEMMA 1: The Regional Vote Shares of Incumbents' Parties**

$$V_{j,d}^A(f_{j,d}^A) = \begin{cases} (f_{j,d}^A + \bar{\sigma}_j) & \text{if } f_{j,d}^A < 1 - \bar{\sigma}_j \\ 1 & \text{if } f_{j,d}^A \geq 1 - \bar{\sigma}_j \end{cases}; \quad (4)$$

$$V_{j,d}^B(f_{j,d}^B) = \begin{cases} (f_{j,d}^B - \underline{\sigma}_j) & \text{if } f_{j,d}^B < 1 + \underline{\sigma}_j \\ 1 & \text{if } f_{j,d}^B \geq 1 + \underline{\sigma}_j \end{cases}. \quad (5)$$

Naturally, party  $P$ 's vote share in region  $j$  increases with  $f_{j,d}^P$ , the regionally targeted efforts of its legislative incumbent. These vote shares are also affected by the partisan biases of regional voters. For equal allocations of  $f_{j,d}^A$ , the vote share  $V_{j,d}^A(\cdot)$  will be higher in regions more biased towards party  $A$  (i.e. with *higher* values of  $\bar{\sigma}_j$ ). Similarly, for equal allocations of  $f_{j,d}^B$  the vote share  $V_{j,d}^B(\cdot)$  will be higher in regions more biased

towards party  $B$  (i.e. with *lower* values of  $\underline{\sigma}_j$ ). Finally, in regions whose incumbent is affiliated with party  $A$  ( $B$ ), any constituency effort at or above the level  $f_{j,d}^A = 1 - \bar{\sigma}_j$  ( $f_{j,d}^B = 1 + \underline{\sigma}_j$ ) yields a vote share of  $V_{j,d}^A(\cdot) = 1$  ( $V_{j,d}^B(\cdot) = 1$ ). FIGURES 3a and 3b display these vote shares visually for regions whose incumbents are from  $A$  and  $B$  respectively.

**FIGURE 3. Regional Vote Shares of Incumbents' Parties**

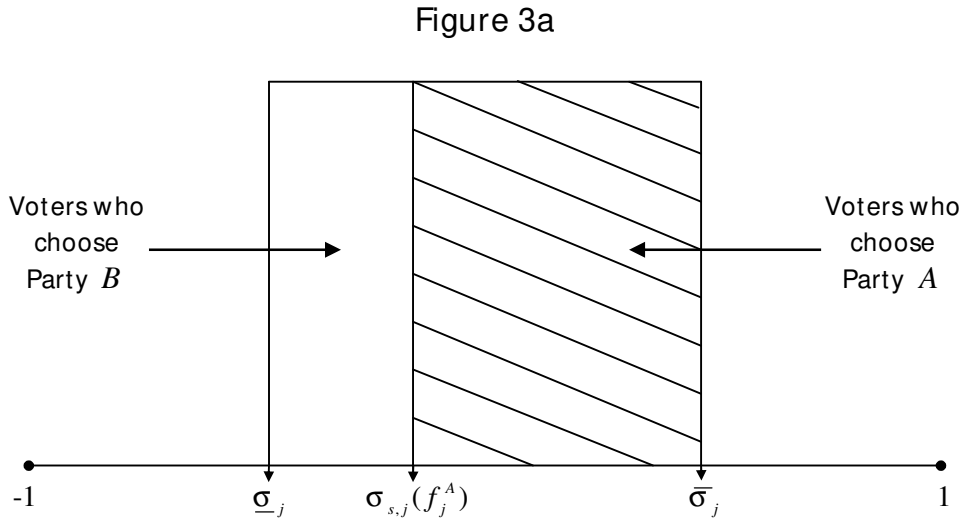
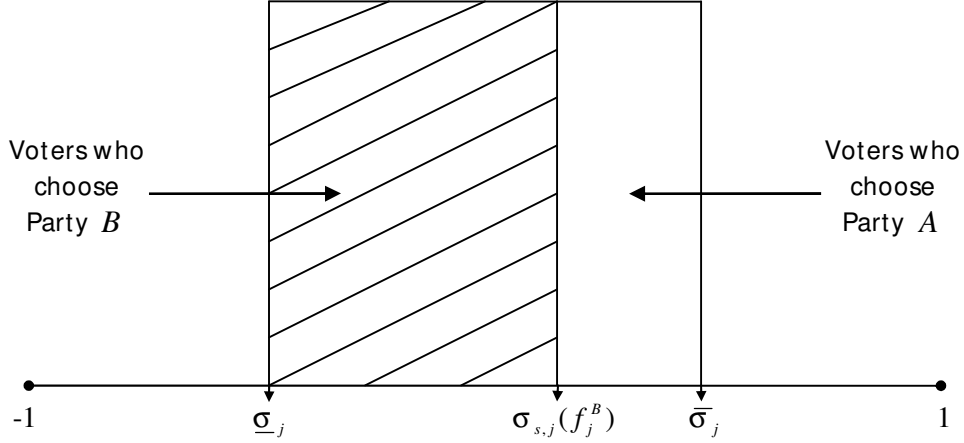




Figure 3b



\* Given some allocation  $f_{j,d}^A$  by an incumbent from party  $A$  in region  $j$ , the expression  $\sigma_{s,j}(f_{j,d}^A)$  represents the partisan attitude of the regional voter whose utility just reaches the reservation level (i.e. for whom  $u_{i,j}^A(\cdot) = \eta = 0$ ; see Appendix A). As such, the utility of regional voters with partisan attitudes  $\sigma_{i,j} > \bar{\sigma}_{s,j}(f_{j,d}^A)$  will surpass the reservation level, and they will vote for party  $A$  (the shaded area in FIGURE 2a).

\* The same applies in regions whose incumbent is from party  $B$ , except that it will be regional voters with partisan attitudes  $\sigma_{i,j} < \sigma_{s,j}(f_{j,d}^B)$  whose utility will surpass the reservation level, and who will thus vote for party  $B$  (the shaded area in FIGURE 2b).

What about the behavior of voters who are not sufficiently satisfied by  $f_{j,d}^P$  and  $\sigma_{i,j}$  to choose the party of their regional incumbent (i.e. for whom  $u_{i,j}^P < 0$ )? In the text I will assume that voters cannot abstain, such that those voters for whom  $u_{i,j}^P < 0$  will simply choose the opposing party (Appendix F introduces abstention). As a result,  $A$ 's vote percentage in regions where the incumbent is from  $B$  will simply be  $[1 - V_{j,d}^B(\cdot)]$ , and  $B$ 's vote percentage in regions whose incumbent is from  $A$  will be  $[1 - V_{j,d}^A(\cdot)]$ .

Incumbents must thus allocate their single unit of effort between  $f_{j,d}^P$  and  $c_{j,d}^P$  in the game's first stage so as to maximize their utility, taking into account the resulting vote outcomes in the game's subsequent electoral stage. In FPTP systems, incumbents can choose an optimal

mix of  $f_{j,d}^P$  and  $c_{j,d}^P$  *decision-theoretically*:

$\mathbf{F}_d = f_{1,d}^P$  is a one-dimensional vector, and effort allocations in one district thus have no bearing on other incumbents' likelihood of gaining re-election.<sup>27</sup> Since only two parties compete, by the definition of plurality rule incumbents must secure *just over half* of their district's votes to gain re-election with certainty (ties are broken randomly). Define  $\hat{f}_{1,d}^P$  as the *critical level* of constituency effort an incumbent from district  $d$  must exert so as to win with certainty. From Lemma 1 above, it is straight-forward to derive these critical levels:

<sup>27</sup> Things change in Section VII: effort that incumbents devote to developing and implementing national-level policies may influence vote shares outside of their own district.

$$\hat{f}_{1,d}^A = \begin{cases} \frac{1}{2} - \bar{\sigma}_j + \varepsilon & \text{if } \bar{\sigma}_j \leq \frac{1}{2} \\ 0 & \text{if } \bar{\sigma}_j > \frac{1}{2} \end{cases} \quad \text{and} \\ \hat{f}_{1,d}^B = \begin{cases} \frac{1}{2} + \underline{\sigma}_j + \varepsilon & \text{if } \underline{\sigma}_j \geq -\frac{1}{2} \\ 0 & \text{if } \underline{\sigma}_j < -\frac{1}{2} \end{cases}, \quad (6)$$

where  $\varepsilon \rightarrow 0$  represents the infinitesimal effort increment needed to make  $V_{1,d}^P(\cdot) > \frac{1}{2}$  (Appendix B addresses the *open-set problem* associated with infinitesimal actions). In districts whose incumbent is from party A (B), if partisan bias exceeds  $\bar{\sigma}_j > \frac{1}{2}$  ( $\underline{\sigma}_j < -\frac{1}{2}$ ) then the

## LEGISLATIVE EQUILIBRIUM UNDER CLPR SYSTEMS

Legislative choice in FPTP systems was decision-theoretic. On the other hand, in PR systems electoral districts send  $M_d > 1$  incumbents to the national Legislature, and  $\mathbf{F}_d \equiv \{f_{1,d}, f_{2,d}, \dots, f_{M_d,d}\}$  is a multi-dimensional strategy vector. In turn, incumbents' probability of winning  $\pi_{j,d}^P(\mathbf{F}_d)$  will be determined not only by their own effort allocation  $f_{j,d}^P$ , but also by

**PROPOSITION 1: In FPTP elections, if  $E^P \geq \hat{f}_{1,d}^P$  then:**

$$f_{1,d}^{P*} = \begin{cases} \hat{f}_{1,d}^P & \text{if } \beta_{1,d}^P > \hat{f}_{1,d}^P \\ 0 & \text{if } \beta_{1,d}^P < \hat{f}_{1,d}^P \\ \hat{f}_{1,d}^P \text{ or } 0 & \text{if } \beta_{1,d}^P = \hat{f}_{1,d}^P \end{cases}$$

district's voters will provide the incumbent majority support even *without* the benefit of constituency effort. Define  $f_{1,d}^{P*}$  as the utility-maximizing choice of the incumbent from district  $d$ . Returning to expression (1) above we see that, trivially, if  $E^P < \hat{f}_{1,d}^P$  then  $f_{1,d}^{P*} = 0$ , i.e. an incumbent without the resources necessary to gain re-election will simply shirk. Proposition 1 identifies  $f_{1,d}^{P*}$  when  $E^P \geq \hat{f}_{1,d}^P$ .

Not surprisingly, the higher the benefit associated with re-election, and the lower its costs in terms of regional effort, the more likely incumbents will choose re-election over the pursuit of personal enrichment.<sup>28</sup>

the effort allocations of the remaining  $(M_d - 1)$  incumbents from their particular multi-member district. Thus, solving for optimal legislative effort allocations in PR systems becomes a *game theoretic* problem. Define  $\bar{A}_d$  and  $\bar{B}_d$  as the number of *current incumbents* from district  $d$  who are affiliated with parties A and B

material interests (such that  $c_{1,d}^P = 1 - \hat{f}_{1,d}^P$ ). Were incumbent legislators to forgo re-election, the optimal allocation would be to choose  $c_{1,d}^P = 1$ . A straight-forward utility comparison tells us that  $(1 - \hat{f}_{1,d}^P) + \beta_{1,d}^P$  is greater than 1 as long as  $\beta_{1,d}^P > \hat{f}_{1,d}^P$ . Incumbents for whom  $\beta_{1,d}^P < \hat{f}_{1,d}^P$  will thus prefer to devote their entire unit effort to personal enrichment. In this case, the district's seat transfers to a non-incumbent from the opposing party, who is assigned no strategic move in the game. Finally, if  $\beta_{1,d}^P = \hat{f}_{1,d}^P$  the incumbent from  $d$  will be indifferent between choosing  $\hat{f}_{1,d}^P$  and choosing  $c_{1,d}^P = 1$ .

<sup>28</sup> To prove this result, first note that the utility incumbents receive from choosing  $\hat{f}_{1,d}^P$  is  $(1 - \hat{f}_{1,d}^P) + \beta_{1,d}^P$ , i.e. they receive with certainty the fixed benefit associated with re-election and devote any surplus effort to pursuing personal

respectively. Given a set of effort allocations  $\mathbf{F}_d$  by district-incumbents in the game's first stage we then can derive  $\mathbf{v}_d^A(\mathbf{F}_d)$ , party  $A$ 's percentage of district  $d$ 's total votes in the game's subsequent electoral stage:

$$\mathbf{v}_d^A(\mathbf{F}_d) = \frac{\sum_{A_d} V_{j,d}^A(\cdot) + \sum_{B_d} [1 - V_{j,d}^B(\cdot)]}{M_d}. \quad (7)$$

Recalling our derivation of  $V_{j,d}^P(\cdot)$  in Lemma 1 above, the first term in (7)'s numerator represents the summation of party  $A$ 's vote shares in regions whose incumbent is from party  $A$ ; and the second term represents the summation party  $A$ 's vote shares in regions whose incumbent is from party  $B$ . These additive terms must then be divided by the district's magnitude to generate an aggregate vote percentage. Party  $B$ 's district-level vote share can be expressed similarly, and ends up being equal to  $\mathbf{v}_d^B = (1 - \mathbf{v}_d^A)$ .

I employ a *simple quota* and *largest remainder rule* to model the process by which these district level vote shares are translated into legislative seats. Define  $q = 1/M_d$  as the electoral quota needed to earn an individual legislative seat, and consider a district of magnitude  $M_d = 10$ , such that  $q = 10\%$ . As an example, if party  $A$  secures a district-level vote share of  $\mathbf{v}_d^A = 58\%$  party  $B$  receives  $\mathbf{v}_d^B = 42\%$ , then  $A$ 's vote share contains 5 full quotas and  $B$ 's vote share contains 4 full quotas, implying that in a first allocation parties  $A$  and  $B$  will receive 5 and 4 seats respectively. As for the final seat, it will go to  $A$  because her *remainder* of 8%, the vote share left over after her 5 quotas are subtracted from  $\mathbf{v}_d^A$ , is larger than  $B$ 's remainder of 2%. In a final tally  $A$  will thus win 6 seats and  $B$  will win 4. If parties have identical remainders of 5%, the final seat is allocated with an unbiased coin-flip.

At election time both parties present a *list* of  $M_d$  candidates to a district's electorate.<sup>29</sup> Among these candidates are the parties' legislative incumbents from district  $d$  and a set of *non-incumbent candidates*, who once again are assigned no strategic move in the game. Party  $A$ 's ( $B$ 's) list thus contains  $\bar{A}_d$  ( $\bar{B}_d$ ) incumbents and  $M_d - \bar{A}_d$  ( $M_d - \bar{B}_d$ ) non-incumbents. Define  $\mathbf{x}_d^P$  as the number of seats won by party  $P$  in district  $d$  during the game's election. After the election, these  $\mathbf{x}_d^P$  seats are subsequently allocated to the *top  $\mathbf{x}_d^P$  candidates on party  $P$ 's electoral list*. This brings us to a crucial distinction between closed-list and open-list systems. In CLPR systems, a candidate's position on his or her party's electoral list is *fixed prior to the general election*. Voters in CLPR elections do not have the option of expressing support for individual candidates from within a party's list, and have no say over which of a party's candidates are allotted its  $\mathbf{x}_d^P$  seats. In contrast, as modeled below, voters themselves determine intra-party seat-allocations in OLPR systems.

Individual parties in CLPR systems may employ any number of organizational mechanisms to fix candidate list positions prior to a general election. I begin by examining the CLPR game in its simplest form by making the following assumption: incumbent legislators from both parties  $A$  and  $B$  occupy *higher list positions than their respective parties' non-incumbents* (**Assumption 1**). This assumption implies no restriction as to which of a party's incumbents is 1<sup>st</sup> on the list, which is 2<sup>nd</sup>, and so on; it stipulates only that incumbents have more favorable positions than non-incumbents. I now solve the following *district-level game*:

<sup>29</sup> Empirically speaking, parties nearly always field as many candidates in a district as that district has seats, *even if* they do not expect to win the district's entire slate of legislative positions.

- a) in a first stage incumbent list positions in district  $d$  are fixed, such that incumbents occupy higher list positions than non-incumbents;
- b) in a second stage all incumbents from  $d$  simultaneously choose  $f_{j,d}^P$  and  $c_{j,d}^P$ ;
- c) a third-stage election is held in which parties  $A$  and  $B$  receive vote percentages  $\mathbf{v}_d^A$  and  $\mathbf{v}_d^B$  respectively;
- d) parties  $A$  and  $B$  are allocated  $\mathbf{x}_d^A$  and  $\mathbf{x}_d^B$  seats respectively via the quota-remainder rule presented above;
- e) and these seats go to  $\mathbf{x}_d^P$  the candidates with party  $P$ 's highest list positions.

Define  $\mathbf{F}_d^* = \{f_{1,d}^*, f_{2,d}^*, \dots, f_{M_d,d}^*\}$  as a *Nash Equilibrium* strategy vector of regional effort allocations for incumbents from district  $d$ .<sup>30</sup>

When incumbents have higher list positions than non-incumbent candidates, it is straight-forward to show that, given any district-level vote outcomes  $\mathbf{v}_d^A$  and  $\mathbf{v}_d^B$  in the game's electoral stage, at least one of the two parties will have *all of its district-level incumbents re-elected*.<sup>31</sup> Define  $\mathbf{F}_d^0 = \{0, 0, \dots, 0\}$  as the *full-shirking* strategy vector, that at which all incumbents from district  $d$  choose  $c_{j,d}^P = 1$  and devote *no* effort to constituent interests; and let

<sup>30</sup> Note that Nash Equilibrium effort allocations by the  $M_d$  incumbents in district  $d$  do not depend on the effort allocation decisions of the  $(N - M_d)$  incumbents from regions outside of  $d$ . As already noted in footnote 27, things change in Section VII, as effort that incumbents devote to national-level policy-making may influence vote shares outside of their own multi-member electoral district.

<sup>31</sup> For example, consider a district  $d$  in which parties  $A$  and  $B$  hold  $\bar{A}_d = 7$  and  $\bar{B}_d = 3$  current seats. If an election is held in which  $A$  and  $B$  win  $\mathbf{x}_d^A = 4$  and  $\mathbf{x}_d^B = 6$  seats, then all of  $B$ 's incumbents (and 3 of its non-incumbents) secure re-election while only 4 of  $A$ 's incumbents are re-elected.

$P_d^+$  ( $P_d^-$ ) denote the party whose incumbents are (are not) all re-elected when the full-shirking vector  $\mathbf{F}_d^0$  is played. Recalling the utility function specified in (1) above, we know that no incumbent who secures re-election when the full-shirking vector  $\mathbf{F}_d^0$  is played has any incentive to alter his or her effort allocation: they secure re-election despite having chosen  $c_{j,d}^P = 1$ , and any deviation would represent a needless transfer of effort away from the pursuit of personal enrichment. This disincentive to constituency effort applies to all incumbent candidates from  $P_d^+$ , but only to the top  $S_d$  incumbents candidates on  $P_d^-$ 's electoral list, where  $S_d$  denotes the number of 'safe seats' won by  $P_d^-$  at the full-shirking vector  $\mathbf{F}_d^0$ .

What about the decision facing an incumbent from  $P_d^-$  at list position  $(S_d + 1)$ , i.e. the incumbent with the highest list position not to receive one of the party's safe seats, and who is thus 'next in line' for re-election? This *marginal candidate* may have a unilateral incentive to defect from  $\mathbf{F}_d^0$ , if by doing so he or she can push party  $P_d^-$ 's district-level vote share  $\mathbf{v}_d^{P_d^-}$  high enough to secure their party the additional seat needed for his or her re-election. Define  $\hat{f}_{m,d}^{P_d^-}$  as the *critical effort level* of the marginal candidate must exert in her region so as to secure re-election, and  $f_{m,d}^{P_d^*}$  as the marginal candidate's equilibrium choice. I now present the following Theorem, proven in Appendix C:

\* **THEOREM 1:** If incumbents occupy higher list positions than non-incumbents, then the unique CLPR Nash Equilibrium in district  $d$  is  $\mathbf{F}_d^* = \{0, 0, \dots, f_{m,d}^{P-*}, \dots, 0, 0\}$ , where the choice  $f_{m,d}^{P-*} \in \{\hat{f}_{m,d}^{P-}, 0\}$  depends on the marginal candidate's utility for re-election  $\beta_{j,m}^{P-}$ , her effort capacity  $E^{P-}$ , and partisanship levels in her region ( $[\underline{\sigma}_{m,d}, \bar{\sigma}_{m,d}]$ ).

Put simply, in equilibrium *at most one* of the district's incumbent legislators ever devotes *any* effort to securing the interests of regional constituents; and in many cases the CLPR Nash Equilibrium is the full-shirking vector itself (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^0$ ). This is due to the fact that incumbents with favorable list positions can *free-ride* on the regional vote-seeking efforts of fellow incumbents with lower list-positions.<sup>32</sup> The proof in Appendix C demonstrates that, at any vector other than  $\mathbf{F}_d^*$  in Theorem 1, either incumbents high on the list will defect so as to free-ride on their co-partisans' mobilizing efforts; or incumbents low on the list will defect to avoid having their efforts appropriated by those with higher list positions.<sup>33</sup>

<sup>32</sup> Supplemental Appendix S1 generalizes this result to *any* situation in which candidates' list positions are independent of incumbents' effort allocations decisions: regardless of the relative placement of incumbent and non-incumbent candidates, in a game with exogenously determined list positions *at most one of a district's incumbents* devotes positive effort to constituency service. Note that Theorem 1's implications for constituency service, in the aggregate, will depend on the *average magnitude* of a country's electoral districts: in countries with *many small districts*, Theorem 1 is consistent with a non-negligible set of incumbents who devote positive effort to constituency service (see Section VI).

<sup>33</sup> In keeping with the tenor of recent research on intra-party dynamics in specific CLPR systems (e.g. Szwarcberg 2009 on Argentina), Supplemental Appendix S1 analyzes a distinct list formation mechanism, assuming that the incumbent from party  $P$  who devotes the highest level of effort to  $f_{j,d}^P$  receives the party's highest list position, the incumbent from party  $P$  who devotes the second-highest level of effort to  $f_{j,d}^P$  receives the party's second-

## LEGISLATIVE EQUILIBRIUM UNDER OLPR SYSTEMS

In OLPR systems voters must simultaneously express support for a political party *and* for a particular candidate from within that political party's list. To capture this mechanism in our present formal context,  $V_{j,d}^P(\cdot)$  will therefore represent not only the percentage of voters from region  $j$  who contribute to party  $P$ 's district-level total, but also the percentage of regional voters who cast individual *candidate votes* in support of the region's incumbent. In pure OLPR systems votes cast against the regional incumbent's party will, by definition, also serve as candidate votes for an individual candidate on the opposing party's list. Recall that both parties field a full slate of  $M_d$  candidates on their electoral lists in district  $d$ , a slate which includes both legislative incumbents and non-incumbent candidates. In keeping with the analysis above, we will assume that in each region whose incumbent is from party  $A$  ( $B$ ), there exists a non-incumbent candidate from party  $B$  ( $A$ ) who amasses the candidate votes of dissatisfied voters. For example, in a region whose incumbent is from party  $A$ ,  $V_{j,d}^A(\cdot)$  will represent the percentage of candidate votes received by the regional incumbent; and  $[1 - V_{j,d}^A(\cdot)]$  will represent the percentage of candidate votes accrued by

highest list position, and so on (see Supplemental Appendix S1). This distinct list-formation mechanism still fails to generate the incentives for constituency service across *all* incumbents that often characterize OLPR systems.

one of party  $B$ 's non-incumbent candidates.<sup>34</sup>

If party  $P$  wins some number  $\mathbf{x}_d^P$  seats in district  $d$  as a result of the game's election, these seats go to the  $\mathbf{x}_d^P$  candidates with the *highest candidate vote scores*. I will assume that candidate vote ties between 2 or more incumbent candidates in a district  $d$  are decided randomly and without bias; and that candidate vote ties between incumbent and non-incumbent candidates are decided in favor of incumbents.<sup>35</sup> I now derive Nash Equilibrium outcomes for the following OLPR game:<sup>36</sup>

- a) in a first stage all incumbents from  $d$  simultaneously choose  $f_{j,d}^P$  and  $c_{j,d}^P$ ;
- b) a second-stage election is held in which incumbent candidates receive a candidate vote share of  $V_{j,d}^P(\cdot)$  in their respective regions, and parties  $A$  and  $B$  receive district-level vote shears  $\mathbf{v}_d^A$  and  $\mathbf{v}_d^B$  respectively;

- c) parties  $A$  and  $B$  are allocated  $\mathbf{x}_d^A$  and  $\mathbf{x}_d^B$  seats respectively via the quota-remainder rule presented above;
- d) and these seats go to the  $\mathbf{x}_d^P$  candidates with  $P$ 's highest candidate vote totals.

Before proceeding I introduce one final piece of notation. For a region  $j$  in district  $d$  whose current incumbent is affiliated with party  $P$ , define  $l_{j,d}^P$  as the percentage of regional *party loyalists*: voters whose value of  $\sigma_{i,j}$  is such that they will choose the party of their regional incumbent *even if* this incumbent chooses  $f_{j,d}^P = 0$ . In regions where the incumbent is from party  $A$  ( $B$ ), loyalists are thus voters for whom  $\sigma_{i,j} > 0$  ( $\sigma_{i,j} < 0$ ).<sup>37</sup> This Section and Appendix D investigate a stylized game in which the number of party loyalists  $l_d \in [0,1]$  is identical across all regions in a particular electoral district  $d$  (**Assumption 2**); and in which both  $\beta_{j,d}^P \geq 1$  for all incumbents and  $E^P \geq 1$  for both parties (**Assumption 3**). Supplemental Appendix S2 extends the model to situations in which  $l_d$  varies across both regions and parties, in which  $\beta_{j,d}^P < 1$  for some or all incumbents, and in which  $E^P < 1$  for one or both parties.

A district's partisanship is influential in defining its incumbents' Nash Equilibrium effort allocations. Proposition 2a in Appendix D shows that, when  $l_d$  is unusually high, incumbent legislators can count on enough support from their party's loyal partisans to eliminate the need for constituency service, and the full-shirking

<sup>34</sup> I have also studied the OLPR game under a distinct assumption: all voters in a district  $d$  who are dissatisfied with party  $P$  transfer their candidate votes to *the same* non-incumbent candidate on the opposing party's list, regardless of the region  $j$  in which they reside. Under this distinct assumption the OLPR game generates *even higher* levels of constituency service than it does under the assumption described above, thus amplifying the model's basic comparative static implications.

<sup>35</sup> The latter assumption is purely expository, and serves to eliminate the *open-set problem* which often arises when strategic actors have continuous action sets. The game is generalizable to situations in which ties between incumbents and non-incumbents are also decided randomly and without bias (Appendix D).

<sup>36</sup> Once again, Nash Equilibria in district  $d$  are unaffected by the behavior of incumbents from regions outside  $d$ , although this changes in Section VII (see ftns 27 and 30 above).

<sup>37</sup> For example, in a region  $j$  with partisan support set  $[-\frac{3}{4}, \frac{1}{4}]$  and whose incumbent is from  $A$ , the loyalist percentage will be  $l_{j,d}^A = \frac{1}{4}$ . Similarly, in a region with partisan support set  $[-\frac{3}{8}, \frac{5}{8}]$  and whose incumbent is from  $B$ , the loyalist percentage will be  $l_{j,d}^B = \frac{3}{8}$ .

vector  $\mathbf{F}_d^0$  is the district-level Nash Equilibrium ( $\mathbf{F}_d^* = \mathbf{F}_d^0$ ). However, at lower levels of party loyalty, incumbents desirous of re-election must devote effort to  $f_{j,d}^P$  so as to *offset the candidate vote totals of non-incumbent candidates*. For example, consider a district  $d$  in which parties  $A$  and  $B$  have  $\bar{A}_d = 7$  and  $\bar{B}_d = 3$  current incumbents respectively, and let district  $d$ 's loyalty level be  $l_d = \frac{1}{4}$ . If  $\mathbf{F}_d^0$  is played, party  $A$ 's district-level vote share is equal to  $\mathbf{v}_d^A = [\frac{1}{4} \cdot (7) + \frac{3}{4} \cdot (3)] / 10 = 40\%$ , and party  $B$ 's vote share is  $\mathbf{v}_d^B = 60\%$ . By the quota-remainder rule  $A$  and  $B$  thus each receive  $\mathbf{x}_d^A = 4$  and  $\mathbf{x}_d^B = 6$  seats.

Furthermore, these legislative seats are allocated almost exclusively to non-incumbent rather than incumbent candidates. To see this, note that at  $\mathbf{F}_d^0$  all incumbents choose  $f_{j,d}^P = 0$  and receive only  $V_{j,d}^P(\cdot) = \frac{1}{4}$  candidate votes, while all non-incumbents receive  $[1 - V_{j,d}^P(\cdot)] = \frac{3}{4}$  candidate votes. Since non-incumbents receive more candidate votes than incumbents, in the game's final stage party  $B$ 's 6 seats will be allocated to 6 of its 7 non-incumbent candidates, and party  $A$ 's 4 seats will be allocated to its 3 non-incumbents and 1 of its incumbent candidates. Given the above assumption that ties between 2 or more incumbents with identical candidate vote totals are randomly decided without bias, when  $\mathbf{F}_d^0$  is played, each of  $A$ 's 7 incumbents has a probability  $\pi_{j,d}^A = \frac{1}{7}$  of gaining the single seat allocated to an incumbent candidate.

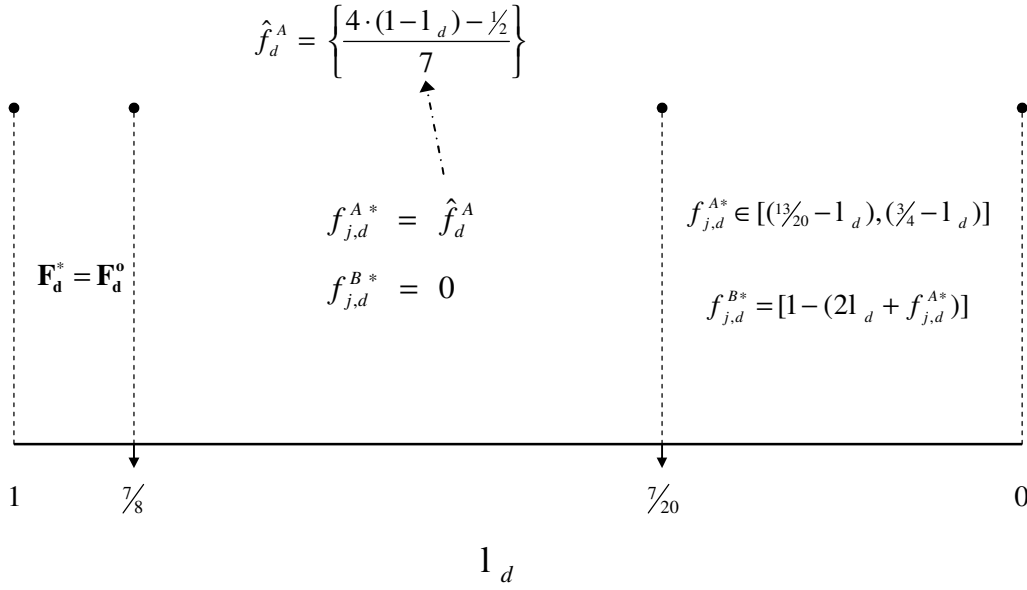
The full-shirking vector  $\mathbf{F}_d^0$  will thus not be a Nash Equilibrium. For example, any one of  $A$ 's 7 incumbents could then choose  $f_{j,d}^A = \varepsilon$  ( $\varepsilon \rightarrow 0$ ), increase her candidate vote total to just above that received by her fellow incumbents, and gain this individual seat with certainty (i.e.

make  $\pi_{j,d}^A = 1$ ). In turn, another of  $A$ 's incumbents could choose  $f_{j,d}^A = \varepsilon'$  ( $\varepsilon' > \varepsilon$ ) and gain the seat with certainty. But then a 3<sup>rd</sup> incumbent could do the same, and so on. Thus, party  $A$ 's 7 incumbents will jockey among themselves over the single legislative seat not allocated to  $A$ 's non-incumbent candidates. In addition to increasing one's own candidate vote total, this jockeying has two important strategic effects: a.) it *increases*  $A$ 's district vote share  $\mathbf{v}_j^A$ ; and b.) it *decreases* the number of preference votes  $[1 - V_{j,d}^A(\cdot)]$  received by the party  $B$ 's non-incumbent candidate in the same region. Thus, while an incumbent's quest for candidate votes emerges for purely competitive reasons, it also has certain 'positive externalities' for fellow incumbents from both parties.

I refer the interested reader to Appendix D, which derives these dynamics' equilibrium consequences for any status quo incumbency pattern and any partisanship level. So as to present intuitively the Nash Equilibrium properties of OLPR competition, FIGURE 4 uses the illustrative example of a district in which parties  $A$  and  $B$  have  $\bar{A}_d = 7$  and  $\bar{B}_d = 3$  current incumbents.

FIGURE 4's x-axis plots values of  $l_d \in [0,1]$  in descending order from left to right. The explicitly marked values of  $l_d$  represent points at which the OLPR game's equilibrium properties change. As already noted, at particularly high values of party loyalty ( $l_d \geq \frac{7}{8}$ ) the OLPR Nash Equilibrium is simply the full-shirking vector ( $\mathbf{F}_d^* = \mathbf{F}_d^0$ ): at this outcome, parties  $A$  and  $B$  win  $\mathbf{x}_d^A = 7$  and  $\mathbf{x}_d^B = 3$  seats respectively by the quota remainder rule, and these seats are allocated to the parties' respective incumbent candidates, since their candidate vote totals outpace those of non-incumbent candidates.

FIGURE 4. OLPR Nash Equilibria



At intermediate values of party loyalty ( $\frac{7}{8} > l_d > \frac{7}{20}$ ), in equilibrium *all* minority party incumbents (i.e. those from party *B*) continue to choose  $f_{j,d}^{B*} = 0$ , but *all* majority party incumbents (i.e. those from party *A*) choose  $f_{j,d}^{A*} = \hat{f}_d^A$ , whose value is explicitly defined in the Figure itself, and once again all incumbents from both parties are re-elected.<sup>38</sup> At lower levels of party loyalty ( $\frac{7}{20} > l_d$ ) the Nash Equilibrium to the OLPR game is no longer unique, although the possible equilibrium outcomes occupy a narrowly defined range of incumbents' action spaces: majority party incumbents may choose  $f_{j,d}^{A*}$  from the range defined in the Figure itself, and minority party incumbents choose a value  $f_{j,d}^{B*}$  which corresponds to this particular value of  $f_{j,d}^{A*}$ . At these outcomes the parties once again win  $\mathbf{x}_d^A = 7$  and  $\mathbf{x}_d^B = 3$

seats respectively; and incumbents' candidate vote shares *perfectly balance* the candidate vote shares of incumbents from the opposing party (i.e.  $V_{j,d}^A = 1 - V_{j,d}^B$ ), such that all incumbents receive exactly as many candidate votes as their party's non-incumbent candidates, and are thus all re-elected.<sup>39</sup>

## INSTITUTIONAL COMPARATIVE STATICS

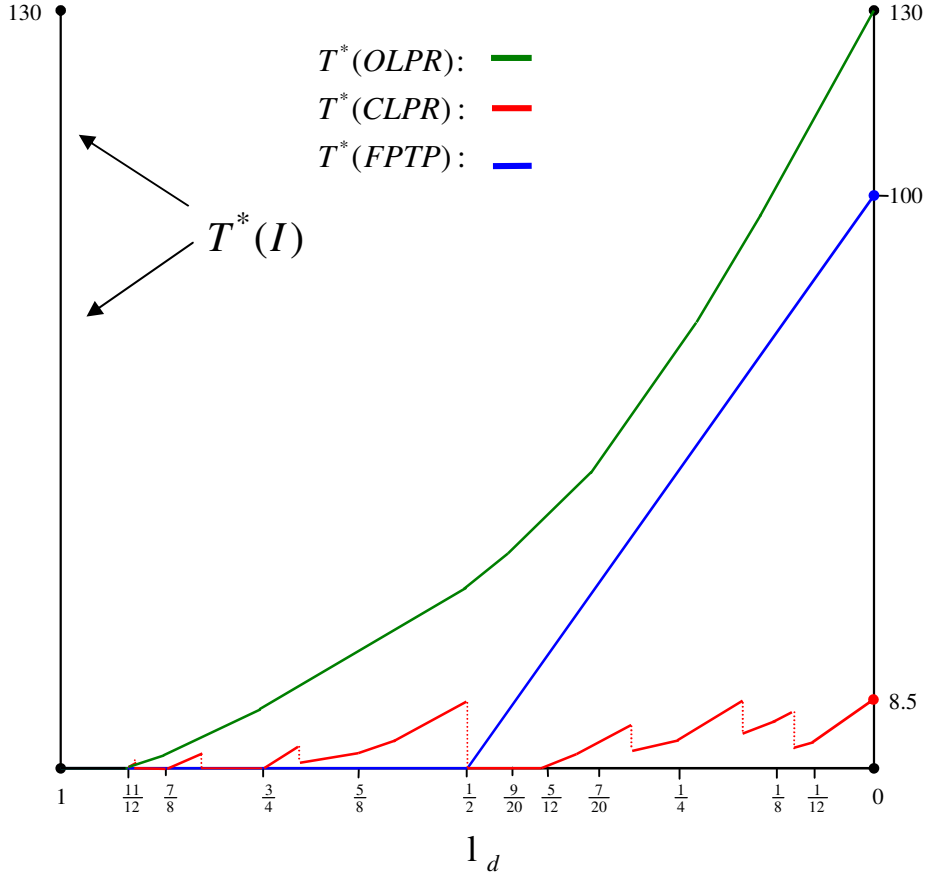
The theoretical results in the preceding Sections apply to individual electoral districts. I will employ a two-stage process to identify the aggregate amount of constituency service generated by a particular electoral system over an *entire Legislature*, first deriving equilibrium outcomes in all of a system's individual electoral districts, and then aggregating these effort allocations across all incumbents. For an institution

<sup>38</sup> The effort level  $\hat{f}_d^A$  represents the level of effort which, when chosen by all incumbents from *A*, pushes this party's vote share *just high enough* to win back  $\mathbf{x}_d^A = 7$  seats. Put otherwise, when all incumbents from *A* choose  $\hat{f}_d^A$ , they *split evenly* the cost, in terms of constituency effort, or re-electing the entire party.

<sup>39</sup> As demonstrated in Supplemental Appendix S2, when the re-election utilities  $\beta_{j,d}^P$  and the effort capacity of *majority party* incumbents (in this case  $E^A$ ) drop below a certain threshold, OLPR Nash Equilibria retain parallel properties, but some subset of incumbents will no longer gain re-election in equilibrium. Finally when  $\beta_{j,d}^P$  is unusually low and/or the majority party's effort constraint is unusually restrictive, the OLPR game *may* have no equilibrium.



FIGURE 5. Aggregate Constituency Effort



$I \in \{FPTP, CLPR, OLPR\}$ , define aggregate constituency service as:

$$T^*(I) = \sum_{P \in \{A, B\}} f_{j,d}^{P*} \quad (8)$$

This is simply the sum of all constituency effort exerted, in equilibrium, by incumbents from both parties. Although I will generate  $T^*(I)$  in a variety of exogenous contexts, to make the analysis tractable I will impose a series of parametric constraints, none of which restricts the results' generality. Firstly, the following computations assume that levels of party loyalty are identical across all of a country's  $N$  regions; define this uniform level of partisanship as  $l \in [0,1]$ . They also assume that  $\beta_{j,d}^P \geq 1$  for all incumbents and that  $E^P \geq 1$  for both parties.<sup>40</sup> Finally, in

situations where OLPR Nash Equilibria are not unique, I analyze the equilibrium whose district-level constituency effort represents the *mean* of all possible equilibria (see Appendix E).

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unusually low values of  $\beta_{j,d}^P$  the OLPR game may under certain circumstances have no NE. That said, this absence of NE does not in fact violate the basic comparative static hypotheses presented below: at these extremely low values of  $\beta_{j,d}^P$ , neither FPTP nor CLPR systems generate any constituency service (i.e. at these extremely low re-election utilities  $T^*(FPTP) = T^*(CLPR) = 0$ ; see Supplemental Appendix S2). As such, at these very low values of  $\beta_{j,d}^P$ , the fact that OLPR competition does not generate a stable outcome in fact makes it more constituency oriented than either FPTP or CLPR systems, which generate stable outcomes characterized by the categorical absence of constituency service.

<sup>40</sup> As discussed in Appendix E, and demonstrated in Supplemental Appendix S2, at

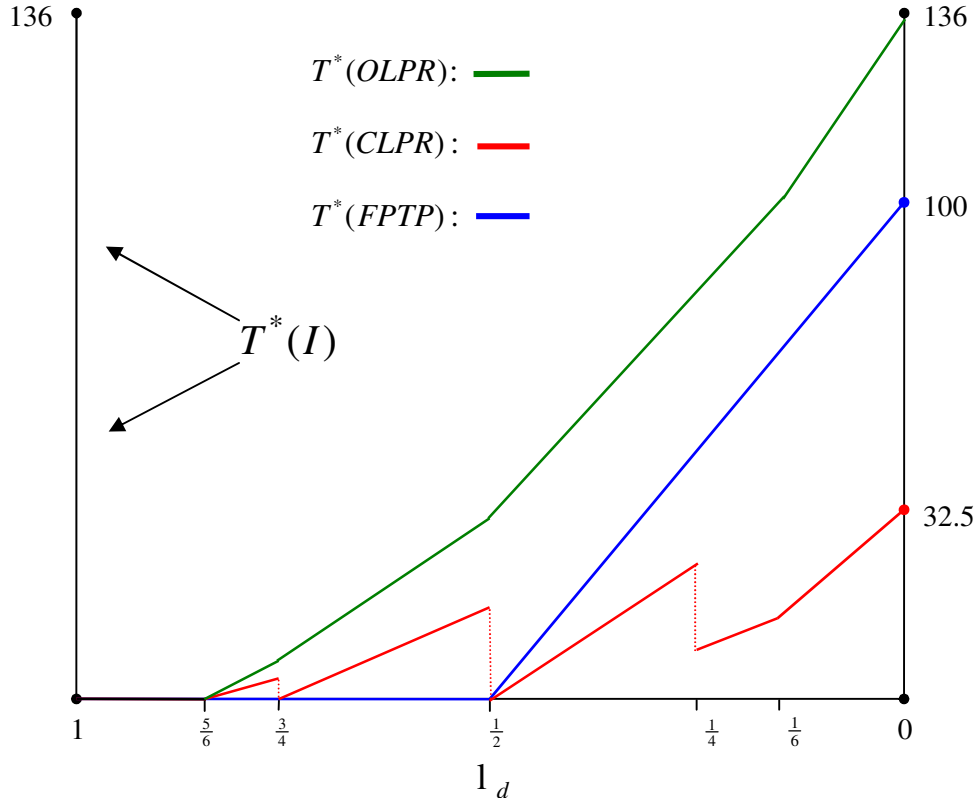
Consider a generic Legislature of  $N = 200$  seats. In FPTP systems the 200 incumbents represent single-member districts. In PR systems, begin with a case in which these 200 seats are divided into  $D = 20$  distinct regions, each with a magnitude of  $M_d = 10$ , with an incumbency status quo in which party  $A$  has a slim 103-to-97 legislative majority. Appendix E contains a more detailed description of the parties' respective incumbency statuses on a district-by-district basis; and presents in detail the two-step process by which  $T^*(I)$  is calculated. FIGURE 5 plots values of  $T^*(I)$  for all three institutions at all possible values of  $l \in (0,1)$ , which move in descending order from left to right on the Figure's x-axis.

The explicitly marked values of  $l$  represent key points of inflection on at least one of the  $T^*(I)$  plots. The first thing to note is that, aside from situations of unusually high party loyalty ( $l > \frac{11}{12}$ ), *OLPR always generates higher levels of constituency service than both FPTP and*

*CLPR*. Secondly, the relationship between  $T^*(CLPR)$  and  $T^*(FPTP)$  varies according to levels of party loyalty. For  $l > \frac{1}{2}$ , CLPR may at times generate slightly higher levels of constituency service than FPTP, although neither institution generates much constituency service to speak of. Once loyalty levels move below  $l < \frac{1}{2}$ , FPTP very quickly outpaces CLPR in generating particularistic effort. Indeed, CLPR in this simulation exhibits a global absence of constituency service incentives.

FIGURE 6 again examines a Legislature of size  $N = 200$  in which party  $A$  again has a slim legislative majority, but this time assumes that PR systems are composed of 40 districts of size  $M_d = 3$  and 40 districts of size  $M_d = 2$ , such that the average district magnitude is significantly lower than that of the previous simulation; district-by-district incumbency details and derivations are again contained in Appendix E.

**FIGURE 6. Aggregate Constituency Effort (cntd)**



Once again, except at the very highest levels of partisanship, OLPR outpaces both CLPR and FPTP in generating constituency service. However, smaller districts have the effect of *amplifying* the distinction between CLPR and FPTP when  $l > \frac{1}{2}$  and *dulling* this distinction when  $l < \frac{1}{2}$ : at higher levels of party loyalty CLPR now *significantly* outperforms FPTP in generating constituency service, while at lower levels CLPR no longer lags as far behind FPTP as in the previous simulation. As demonstrated in Appendix E, these institutional comparative statics do not depend on the legislative status quo: regardless the district-by-district incumbency breakdown, OLPR outperforms both FPTP and CLPR in generating aggregate-level constituency service at all but the highest levels of electoral partisanship (in which case  $T^*(I) = 0$  for all three institutions); and at higher (lower) levels of partisanship CLPR (FPTP) generates greater aggregate constituency service than FPTP (CLPR).<sup>41</sup>

## CONCLUDING DISCUSSION

This paper's model thus suggests that OLPR will, under almost all circumstances, generate greater aggregate constituency service than its CLPR and FPTP counterparts, while the latter two systems' relative performance depends on an electorate's partisanship and the size of electoral districts (as well as the particular candidate nomination procedures used in CLPR systems: see fn 33 and fn 41). Given our theoretical primitives, the reciprocal comparative static implication is

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<sup>41</sup> Supplemental Appendix S1 also undertakes a third simulation which employs the distinct assumption, described in fn 33, that the more effort an incumbent devotes to  $f_{j,d}^P$  the higher he or she will be placed on party  $P$ 's electoral list. Under this list-formation mechanism, OLPR continues to outperform both CLPR and FPTP in generating constituency service, although the *range of partisanship values over which OLPR outpaces CLPR shrinks*; and once again CLPR generates significantly more aggregate constituency service than FPTP systems at higher levels of partisanship, while the distinction between CLPR and FPTP at lower levels of partisanship diminishes further.

that OLPR systems should generate *lower levels of political corruption* than their CLPR and FPTP counterparts. As such, the theoretical results presented in Sections III-VI match quite nicely the empirical results presented in Section II. One might object that the inverse relationship between constituency service and legislative corruption is built into this model *by fiat*, i.e. all legislative effort not devoted to constituency service is, *by construction*, devoted to personal material enrichment. In a previous working paper (Kselman 2008b), whose explicit results I don't present here for reasons of space, this paper's model is embedded in a larger game which includes a first move by an *incumbent Prime Minister* (PM), such that the legislative equilibria derived here become *subgames*.

The incumbent PM must allocate a fixed amount of public effort between targeted transfer policies, universalistic public good policies, and personal material enrichment, taking into account the downstream incentives of incumbent legislators. In turn, legislative incumbents (from both parties...) may devote effort to supporting the PM's particularistic and/or universalistic policy proposals; or to pursuing their own personal material enrichment. In this expanded model the electorate's partisanship, which was exogenous above, becomes partially endogenous to the PM's policy proposals, and the extent to which these proposals find support among legislative incumbents.<sup>42</sup> A crucial parameter in this more complicated game is voters' relative *receptiveness* (i.e. elasticity) to particularistic as opposed to universalistic policy proposals. The results complement nicely those derived in Sections III-VI. Aggregate political corruption is *always* lower under OLPR than either CLPR or FPTP; and the latter two systems policy performance depends on the voters' policy elasticities: when voters are highly responsive to universalistic (particularistic) policies,

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<sup>42</sup> There remains an exogenous element, corresponding to voters' affective 'partisan identification', which is independent of short-term policy considerations.

CLPR (FPTP) generates more salutary public policy patterns than FPTP (CLPR).<sup>43</sup>

In addition to calling into question a growing consensus as to the consequences of formal electoral institutions, this paper speaks to the frequent conflation, among both policy and academic circles, of political particularism and corruption. Indeed, critics of clientelism and other forms of targeted public policy often use the two terms interchangeably. As well, it is often suggested that particularism is a precondition for legislative corruption, i.e. that legislators' opportunities for personal material enrichment are particularly strong when public policy is highly targeted. However, a growing body of recent research offers a more nuanced normative and empirical appraisal of particularistic forms of accountability. Keefer and Vlaicu (2008) argue that the targeted public policies often improve aggregate social welfare when politicians cannot credibly commit to the provision of public goods. Fernandez and Pierskalla (2009) find that countries with high levels of political particularism in fact *outperform* their counterparts on select dimensions of economic and human development (e.g. infant mortality and literacy). More generally, an ambitious project by Herbert Kitschelt and collaborators (Freeze *et al.*

2008) aimed at gathering data on alternative forms of political accountability in a sample of 90 contemporary democracies takes as a starting point the distinction between political particularism and corruption and/or other pernicious governance practices. The current paper shares with this research the undercurrent that at times particularistic accountability may serve as a 'second-best' policy alternative when the exogenous environment is not conducive to more normatively palatable forms governance and accountability.

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<sup>43</sup> Specifically, when voters are highly responsive to public good policies, all three electoral systems impel both the incumbent PM and her legislative caucus to devote significant effort to the implementation of national level public goods policies (this universalism is particularly high in CLPR systems), while OLPR continues to generate lower levels of aggregate corruption than both CLPR and FPTP (with FPTP generating the lowest 'working-shirking' ratio). As voters become less responsive to universalistic policies and more responsive to targeted policies, incumbents in all three systems shift effort away from the development and implementation of national-level policies, but the destination of this effort differs across systems: in OLPR systems *most* of this previously universalistic effort is spent on particularistic goods; in FPTP systems *some* of this effort goes to particularism while a non-negligible portion goes to corruption; and in CLPR systems *most* of this effort goes to corruption.

## APPENDIX A. DATA AND MEASUREMENT

\* All of the data in the following table comes directly from the publicly available dataset which accompanies Persson and Tabellini (2003), and which is available at

<http://people.su.se/~tpers/>. All of the individual variable coding descriptions come directly from the data Appendix in Persson and Tabellini (2003).

**TABLE A1. Data from Persson and Tabellini (2003)**

### **Dependant Variable**

-GRAFT: point estimate of 'Graft', the sixth cluster of Kaufman *et al.*'s *Governance Indicators* focusing on perceptions of corruption, with a possible range of 0-to-10, where lower values correspond to better outcomes.

### **Electoral Formula and Ballot Structure**

-PIND: continuous measure of ballot structure, defined as  $[1 - (\text{List Seats}/\text{Total Seats})]$ , where the second term represents the percentage of legislators elected on party lists divided by the total number of seats in the Legislature. As such, PIND measures the percentage of legislators elected independent of party lists.

-PINDO: continuous measure of ballot structure, defined as  $[1 - (\text{Closed List Seats}/\text{Total Seats})]$ , where the second term represents the percentage of legislators elected on *closed party lists* divided by the total number of seats in the Legislature.

-MAJ: dummy variable which equals 1 for country's whose lower house is elected by plurality rule, and equals 0 otherwise.

### **Political-Institutional Controls**

-MAGN: inverse district magnitude, defined as the number of electoral districts inside a particular country divided by the number of seats in the country's Legislature.

-PRES: dummy variable equal to 1 in presidential regimes and 0 otherwise. Only regimes where the confidence of the Assembly is not necessary for the executive are excluded among presidential regimes.

-FEDERAL: dummy variable equal to 1 if the country has a federal political structure and 0 otherwise.

-GASTIL: average of indices for civil liberties and political rights, where each index is measured on a one-to-seven scale with one representing the highest degree of freedom and 7 the lowest.

### **Historical-Cultural Controls**

-AGE: age of democracy, defined as  $(2000 - \text{DEM\_AGE})/200$  and varying between 0 and 1, where the US is the world's oldest democracy with a value of AGE = 1. DEM\_AGE is coded as the first year of democratic rule, corresponding to the first year of an uninterrupted string of positive yearly values on the variable POLITY until the end of the sample.

-COL\_UK: dummy variable equal to 1 if the country was a former British colony and 0 otherwise.

-PROT80: percentage of the population in each country professing the Protestant religion in 1980.

-CATHO80: percentage of the population belonging to the Roman Catholic Church in 1980.

-CONFU: dummy variable equal to 1 if the majority of a country's population is Confucian/Buddhist/Zen, and equal to 0 otherwise.

-AVELF: index of ethno-linguistic fractionalization, approximating the level of ethnic and linguistic fragmentation within a country, ranging from 0 (homogeneous) to 1 (strongly fractionalized) comprising an average of five different indices.

### Socio-Economic Controls

- LPOP: natural log of total population.
- EDUGER: total enrollment in primary and secondary education, as a percentage of the relevant age group in the population.
- LYP: natural log of per capita GDP, where real GDP is defined as per capita GDP in constant dollars expressed in international prices (base year 1985).
- TRADE: sum of exports and imports of goods and services measured as a share of GDP.

### Regional Dummies

- LAAM: regional dummy variable equal to 1 if the country is in Latin America, Central America, or the Caribbean, and equal to 0 otherwise.
- OECD: dummy variable equal to 1 for all countries which were members of the OECD before 1993, and 0 for all other countries (except for Turkey, which is assigned a value of 0 despite having been a member nation prior to 1993).

\* Table A2 contains all countries values on the variables FPTP, CLPR, OLPR, and HYBRID. Countries are placed in four distinct columns depending on their *predominant system* (see text page 8). For countries with mixed systems, their values on the distinct institutional variables are labeled in parentheses. These measures were coded using a variety of different sources for the sake of cross-checking, including but not limited to: Golder (2004); Seddon *et al.* (2002); the data Appendix in Cox (1997); and the Inter-Parliamentary Union's online database, which can be found at <http://www.ipu.org/english/home.htm>.

\* In keeping with the dependent variable's time point, countries are coded according to the electoral system present during the years 1994-1997. Four countries undertook major institutional reforms in 1993: New Zealand went from an FPTP system to a mixed FPTP-PR system in which the upper-tier serves as a *corrective tier* for any disproportionality introduced in the FPTP tier (see discussion of corrective tiers immediately following); Italy went from an OLPR system to mixed FPTP-CLPR system, also with a corrective PR tier; Venezuela went from a pure CLPR system to a mixed FPTP-CLPR system with a corrective upper-tier; and Japan went from using the Single Non-Transferable-Vote in multi-member districts to a mixed FPTP-CLPR system in which the two tiers are

independent (i.e. the PR-tier is not corrective). I have re-run all of the paper's empirical analyses on a sample in which these three cases are coded as intermediate, i.e. their values are weighted equally by the system in place before 1993 and that in place after 1993. The paper's empirical results are completely unaffected. Bolivia and the Philippines both experienced institutional change in 1996, but these changes did not become effective for electoral competition until after 1997.

\* A number of countries which used ostensibly mixed systems are here coded as pure system types: Germany, New Zealand, Italy, Venezuela, and South Korea. This is due to the fact that a parties' seat allocation in one-tier is not *independent* from their performance in the alternative tier (all cases can be recoded as mixed without changing the paper's empirical results). The first four cases use a corrective, national-level PR tier to correct for any disproportionality in vote shares which arise in the lower FPTP tier. Political parties thus have every incentive to engage in vote-seeking *as if* the system were purely proportional, since in the end seats will be allocated on a purely proportional basis. Similarly, the small upper-tier in South Korean elections serves to amplify the seat majority of whichever party wins a plurality of FPTP seats, such that parties' real emphasis will be on the lower tier (i.e. South Korea is coded as pure FPTP).

**TABLE A2. Electoral Formula and Ballot Structure**

<b><u>FPTP</u></b>	<b><u>CLPR</u></b>	<b><u>OLPR</u></b>	<b><u>HYBRID</u></b>
Bahamas	Argentina	Brazil	Australia
Bangladesh	Austria	Chile	Cyprus
Barbados	Belgium	Denmark	Malta
Belize	Bolivia	Czech Republic	Mauritius
Botswana	Bulgaria	Estonia	Taiwan (HYBRID=.58; CLPR=.42)
Canada	Colombia	Finland	Thailand
Fiji	Costa Rica	Greece (OLPR=.94; FPTP=.006)	
France	Dominican Republic	Latvia	
Gambia	Ecuador	Luxemburg	
Ghana	El Salvador	Poland (OLPR=.85; FPTP=.15)	
India	Germany	Slovak Republic	
Jamaica	Guatemala	Sri Lanka	
Japan (FPTP=.6; CLPR=.4)	Honduras	Switzerland (OLPR=.975; FPTP=.025)	
Malawi	Hungary (CLPR=.54; FPTP=.46)		
Malaysia	Iceland		
Mexico (FPTP=.6; CLPR=.4)	Israel		
Nepal	Italy		
Pakistan	Namibia		
Papua New Guinea	Netherlands		
Philippines	New Zealand		
Singapore	Nicaragua		
South Korea	Norway		
St. Vincent	Paraguay		
Trinidad	Peru		
USA	Portugal		
Uganda	Romania		
UK	Senegal (CLPR = .583; FPTP=.417)		
Ukraine	South Africa		
Zambia	Spain		
Zimbabwe	Sweden		
	Turkey		
	Uruguay		
	Venezuela		

## APPENDIX B. REGIONAL VOTE SHARES AND OPEN-SETS

\* In this Appendix, I first derive formal expressions for party  $P$ 's vote share in a region  $j$  whose current incumbent is affiliated with party  $P$  (Lemma 1). These vote share formulae are then employed in the derivation of CLPR and OLPR Nash Equilibria in Appendices C and D.

\* Having derived these vote share formulae, I then address the open-set problem which oftentimes arises when strategic actors have continuous action spaces.

### I. Regional Vote Shares

\* Recalling the notation developed in Section III of the text, I now prove Lemma 1. I begin by proving that, given some effort allocation  $f_{j,d}^A$  by the regional incumbent, Party  $A$ 's vote share in a region  $j$  whose incumbent is from  $A$  is equal to:

$$V_{j,d}^A(f_{j,d}^A) = \begin{cases} [f_{j,d}^A + \bar{\sigma}_j] & \text{if } f_{j,d}^A < 1 - \bar{\sigma}_j \\ 1 & \text{if } f_{j,d}^A \geq 1 - \bar{\sigma}_j \end{cases}. \quad (\text{B1})$$

\* Proof: Given some allocation  $f_{j,d}^A$  by the regional incumbent, define  $\sigma_{s,j}(f_{j,d}^A)$  as the partisan attitude of region  $j$ 's swing voter, i.e. the voter whose utility *exactly* reaches the reservation level  $\eta$ . Recalling the text's specification of voter utility for party  $A$ , this implies:

$$u_{s,j}^A(f_{j,d}^A) = \sigma_{s,j}(f_{j,d}^A) + f_{j,d}^A = \eta. \quad (\text{B2})$$

\* Given some allocation  $f_{j,d}^A$ , voters in region  $j$  with partisan attitude  $\sigma_{i,j} > \sigma_{s,j}(f_{j,d}^A)$  will thus have a higher utility for party  $A$  than the region's swing voter. In turn, the utility of voters in region  $j$  with partisan attitudes in the range  $\sigma_{i,j} > \sigma_{s,j}(f_{j,d}^A)$  will surpass the reservation level, and this subset of regional voters will vote for  $A$ .

\* Given the assumption that partisan preferences in region  $j$  are uniformly distributed, the percentage of region  $j$ 's voters for whom  $\sigma_{i,j} > \sigma_{s,j}(f_{j,d}^A)$  can be

written as follows (see FIGURE 3a for a visual presentation of this percentage):

$$V_{j,d}^A(f_{j,d}^A) = \frac{\bar{\sigma}_j - \sigma_{s,j}(f_{j,d}^A)}{\bar{\sigma}_j - \underline{\sigma}_j}. \quad (\text{B3})$$

\* Recalling from the text that  $\eta = 0$  and  $\bar{\sigma}_j - \underline{\sigma}_j = 1$  by construction (and without loss of generality), we can substitute  $\eta = 0$  into (B2) and  $\bar{\sigma}_j - \underline{\sigma}_j = 1$  into (B3). Then, by using (B2) to substitute into (B3) and rearranging we obtain the following expression:

$$V_{j,d}^A(f_{j,d}^A) = [\bar{\sigma}_j + f_{j,d}^A]. \quad (\text{B4})$$

\* Since  $V_{j,d}^A(f_{j,d}^A)$  cannot be greater than 1, any value of  $f_{j,d}^A$  such that (B4) is greater than 1 implies a vote share of 1 (i.e. devoting more effort than  $f_{j,d}^A = 1 - \bar{\sigma}_j$  is unnecessary to secure 100% voter support in region  $j$ ), thus implying (B1).

\* An identical process yields  $V_{j,d}^B(f_{j,d}^B)$ , party  $B$ 's vote share in a region  $j$  whose current incumbent is affiliated with  $B$  (proof available upon request).

### II. The Open-Set Problem

\* The term  $\varepsilon \rightarrow 0$  appears in my definition of  $\hat{f}_{1,d}^P$  on page 18 of the text, where  $\hat{f}_{1,d}^P$  is defined as the effort level the incumbent from region  $j$  must choose to secure the electoral support of just over half of her region's voters. This term appears because, if the incumbent associated with region  $j$  does not add this infinitesimal increment to the effort allocation  $\hat{f}_{1,d}^P$ , she will receive exactly  $V_{1,d}^P(\cdot) = 1/2$  of the district's vote. In plurality rule elections with only two political parties, receiving  $V_{1,d}^P(\cdot) = 1/2$  implies winning the contest with probability  $\pi_{1,d}^P = 1/2$ . The incumbent from region  $j$  will thus always prefer increasing her regional effort infinitesimally so as to win the seat with certainty.



\* However, in the strictest sense  $\hat{f}_{1,d}^P$  does not in fact exist, since  $\varepsilon$  can always be made infinitesimally closer to 0, i.e. we have what game theorists label an open-set problem. As is often noted, this problem is purely technical, and can be eliminated by assuming that incumbents' action spaces are composed of measurable but minute effort increments. This technicality aside, in the following Appendices I will continue to assume that incumbents' action spaces are continuous so as to avoid added numerical complexity. All results are generalizable to situations with non-continuous action spaces composed of measurable but minute effort increments.

## APPENDIX C. CLPR NASH EQUILIBRIA

\* This Appendix provides an exhaustive proof of Theorem 1 from the text. In turn, Supplemental Appendix S1 generalizes Theorem 1 to any situation in which candidates' list positions are determined exogenously, and then analyzes a game in which incumbents' list positions are determined endogenously by their constituency effort allocations.

\* I now derive NE results of the game whose sequential structure is outlined in page 22 of the text, in which incumbents are assumed to be placed higher on electoral lists than their parties' non-incumbents (**Assumption 1**). Recall from the text that, when the full-shirking vector  $\mathbf{F}_d^o$  is played in district  $d$  and **Assumption 1** is employed, at least one of the two parties has all of its district-level incumbents re-elected. More specifically, when the  $\mathbf{F}_d^o$  is played there are two possibilities:

- a) all incumbent candidates, and at least one non-incumbent candidate, from one party are re-elected, but not all incumbents from the other party are re-elected (see example from the text, fn 31). Define the party whose incumbents are (are not) all re-elected as  $P_d^+$  ( $P_d^-$ ).
- b) all incumbent candidates from both parties are re-elected. This occurs if, when the full-shirking vector  $\mathbf{F}_d^o$  is played, parties  $A$  and  $B$  win  $\mathbf{x}_d^A = \bar{A}_d$  and  $\mathbf{x}_d^B = \bar{B}_d$  seats respectively by the quota-remainder rule.

\* I begin the analysis with Theorem 1 from the text, which applies to districts in which one party has all of its incumbents re-elected if  $\mathbf{F}_d^o$  is played, but the other does not.

### I. Theorem 1: Proof of Existence

\* **Lemma 2:** Any strategy vector  $\mathbf{F}_d$  in which at least one incumbent sets  $f_{j,d}^P > 0$  but *does not* gain reelection is *not* a NE; and no incumbent will ever deviate from

$f_{j,d}^P = 0$  if this deviation does not result in re-election.

\* **Proof:** if  $f_{j,d}^P > 0$  but the incumbent in question does not secure reelection, then she will always prefer deviating and choosing  $f_{j,d}^P = 0$ , since  $U_{j,d}^P(f_{j,d}^P) = (1 - f_{j,d}^P)$  is less than  $U_{j,d}^P(0) = 1$ . For the same reason, deviating from  $f_{j,d}^P = 0$  without winning re-election is strictly-dominated by keeping one's choice at  $f_{j,d}^P = 0$ .

\* None of the incumbents from  $P_d^+$  defect from  $\mathbf{F}_d^o$  in Theorem 1, as they secure reelection without devoting any effort to constituency service: even if the marginal candidate chooses  $\hat{f}_{m,d}^P$  and secures her party  $P_d^-$  one seat more than it gains at the full-shirking vector  $\mathbf{F}_d^o$ , this seat will be taken from one of  $P_d^+$ 's non-incumbent candidates, since incumbents are placed above non-incumbents on their electoral list by **Assumption 1** (and since at least one non-incumbent from  $P_d^+$  is elected when  $\mathbf{F}_d^o$  is played). Similarly, none of the top  $S_d$  incumbents from  $P_d^-$  has any incentive to defect from  $\mathbf{F}_d^o$  in Theorem 1, as they also secure reelection without devoting any effort to constituency service.

\* In order to specify the marginal candidate's equilibrium behavior, we begin by identifying the conditions under which this candidate will have a payoff-enhancing deviation from the full-shirking vector  $\mathbf{F}_d^o$ .

Define  $l_{m,d}^{P-}$  as the number of party loyalists in the marginal incumbent's region. Party loyalists are those regional voters whose partisan bias  $\sigma_{i,j}$  is such that they will choose the party of their regional incumbent even if that incumbent chooses  $f_{j,d}^P = 0$ . Given our distributional assumptions, and in particular the assumption that  $-1 < \underline{\sigma}_j < 0 < \bar{\sigma}_j < 1$  (page 15 of the text), it is straightforward to see that all regions will contain some non-zero

number of party loyalists. This is highly plausible: one can hardly imagine a region in which an incumbent's party would receive a vote share of 0, regardless of her behavior during the previous legislative term.

\* The marginal candidate needs her party's aggregate district vote share to reach the following level in order to gain re-election (at this level her party's electoral remainder just outpaces that of the opposing party  $P_d^+$ , thus securing  $P_d^-$  a total of  $\mathbf{x}_d^{P^-} = (S_d + 1)$  legislative seats):

$$\hat{\mathbf{v}}_d^{P^-}(S_d + 1) = (S_d/M_d) + (1/2M_d) + \varepsilon \quad (\varepsilon \rightarrow 0). \quad (\text{C1})$$

i.) Case 1: Categorical Non-Deviation of the Marginal Candidate

\* Define  $P_d^-$ 's aggregate district vote share when the full-shirking vector  $\mathbf{F}_d^o$  is played as  $\mathbf{v}_d^{P^-}(\mathbf{F}_d^o)$ . If  $\{\mathbf{v}_d^{P^-}(\mathbf{F}_d^o) + [(1 - 1^{P^-}_{m,d})/M_d]\} < \hat{\mathbf{v}}_d^{P^-}(S_d + 1)$ , then even if the marginal candidate deviated from  $\mathbf{F}_d^o$  so as to secure 100% voter support in her district,  $P_d^-$  would still receive only  $\mathbf{x}_d^{P^-} = S_d$  seats, and the marginal candidate would not be re-elected. Put otherwise, there are not enough undecided voters in the marginal incumbent's region to secure  $P_d^-$  an additional seat. In this situation, by Lemma 2 the marginal incumbent has no incentive to defect from  $\mathbf{F}_d^o$ .

ii.) Case 2: Potential Deviation by the Marginal Candidate

\* If  $\{\mathbf{v}_d^{P^-}(\mathbf{F}_d^o) + [(1 - 1^{P^-}_{m,d})/M_d]\} > \hat{\mathbf{v}}_d^{P^-}(S_d + 1)$ , then the marginal candidate may be able to deviate from  $\mathbf{F}_d^o$  so as to secure her own re-election. Recall that  $\hat{f}_{m,d}^{P^-}$  represents the critical level of constituency service the marginal candidate must exert in order to push  $P_d^-$ 's vote total up to  $\hat{\mathbf{v}}_d^{P^-}(S_d + 1)$ , i.e.

the effort necessary to move  $P_d^-$ 's seat total from  $\mathbf{x}_d^{P^-} = S_d$  to  $\mathbf{x}_d^{P^-} = (S_d + 1)$ .<sup>44</sup>

\* In this case, the marginal incumbent will have the incentive to deviate from  $\mathbf{F}_d^o$  and choose  $\hat{f}_{m,d}^{P^-}$  as long as  $\hat{f}_{m,d}^{P^-} < \beta_{m,d}^{P^-} E^{P^-}$ , i.e. as long as both: a.) the payoff from deviating to  $\hat{f}_{m,d}^{P^-}$  and gaining re-election,  $U_{m,d}^{P^-}(\hat{f}_{m,d}^{P^-}) = [(1 - \hat{f}_{m,d}^{P^-}) + \beta_{m,d}^{P^-}]$ , is higher than the payoff accrued from devoting all effort to the pursuit of personal wealth  $U_{m,d}^{P^-}(0) = 1$ ; and b.) the marginal candidate has sufficient effort capacity to secure the necessary votes. On the other hand, if either  $\hat{f}_{m,d}^{P^-} > \beta_{m,d}^{P^-}$  or  $\hat{f}_{m,d}^{P^-} > E^{P^-}$  she will not have the incentive to deviate from  $\mathbf{F}_d^o$ . Finally, if  $\hat{f}_{m,d}^{P^-} = \beta_{m,d}^{P^-}$  but  $\hat{f}_{m,d}^{P^-} \leq E^{P^-}$  then the marginal candidate will be indifferent between deviating to  $\hat{f}_{m,d}^{P^-}$  and remaining at  $f_{m,d}^{P^-} = 0$ .

iii.) Case 3: Potential Deviation by the Marginal Candidate

\* If  $\{\mathbf{v}_d^{P^-}(\mathbf{F}_d^o) + [(1 - 1^{P^-}_{m,d})/M_d]\} = \hat{\mathbf{v}}_d^{P^-}(S_d + 1)$ , then the marginal candidate may be able to deviate from  $\mathbf{F}_d^o$  so as to move her party into a 'remainder tie' with the opposing party: both parties will have identical remainders of 5% if the marginal candidate secures 100% electoral support in her own region. For this particular case, we will thus redefine  $\hat{f}_{m,d}^{P^-} = (1 - 1^{P^-}_{m,d})$  as the critical level of constituency effort necessary to secure the marginal candidate re-election with  $\pi_{m,d}^{P^-} = 1/2$ .

\* In this case, the marginal incumbent will have the incentive to deviate from  $\mathbf{F}_d^o$  and choose  $\hat{f}_{m,d}^{P^-} = (1 - 1^{P^-}_{m,d})$  as long as  $(1 - 1^{P^-}_{m,d}) < 1/2 \cdot \beta_{m,d}^{P^-} E^{P^-}$ , i.e. as long as both: a.) the payoff accrued from deviating

<sup>44</sup> Once again, this critical value does not, technically, exist due to the open-set problem (Appendix B). As discussed above, we could address this trivially by making incumbents' action sets non-continuous.

to  $\hat{f}_{m,d}^{P^-} = (1 - 1_{m,d}^{P^-})$  and gaining re-election with probability  $\pi_{m,d}^{P^-} = 1/2$  is higher than the payoff accrued from devoting all effort to the pursuit of personal wealth  $U_{m,d}^{P^-}(0) = 1$ ; and b.) the marginal candidate has sufficient effort capacity to secure the necessary votes. On the other hand, if either  $(1 - 1_{m,d}^{P^-}) > 1/2 \cdot \beta_{m,d}^{P^-}$  or  $(1 - 1_{m,d}^{P^-}) > E^{P^-}$  she will not have the incentive to deviate from  $\mathbf{F}_d^0$ . Finally, if  $(1 - 1_{m,d}^{P^-}) = 1/2 \cdot \beta_{m,d}^{P^-}$  and  $(1 - 1_{m,d}^{P^-}) \leq E^{P^-}$  then the marginal candidate will be indifferent between deviating and remaining at  $f_{m,d}^{P^-} = 0$ .

#### iv.) Non-Deviation of all Other Candidates

\* We have now established that, as long as all other incumbents choose  $f_{j,d}^P = 0$ , the marginal incumbent's optimal choice  $f_{m,d}^{P-*} \in \{0, \hat{f}_{m,d}^{P-}\}$  will depend on the exogenous parameters  $1_{m,d}^{P-}$ ,  $\beta_{m,d}^{P-}$ , and  $E^{P-}$ . The final step in proving Existence is demonstrating that none of  $P_d^-$ 's candidates at list positions lower than  $(S_d + 1)$  wish to defect from  $\mathbf{F}_d^*$  in Theorem 1. The following expression captures the aggregate vote share  $P_d^-$  needs to gain  $\mathbf{x}_d^{P-} = (S_d + 2)$  legislative seats (at this level, her party's electoral remainder just outpaces that of the opposing party  $P_d^+$ , thus securing  $P_d^-$  a total of  $\mathbf{x}_d^{P-} = (S_d + 2)$  legislative seats):

$$\hat{\mathbf{v}}_d^{P-}(S_d + 2) = (S_d / M_d) + (3 / 2M_d) + \varepsilon \quad (\varepsilon \rightarrow 0). \quad (\text{C2})$$

\* Define  $1_{S_d+2,d}^{P-}$  as the number of party loyalists in the region whose incumbent occupies position  $(S_d + 2)$  on  $P_d^-$ 's electoral list. As well, define  $\mathbf{v}_d^{P-}(\mathbf{F}_d^*)$  as the aggregate district vote share received by  $P_d^-$  when the NE strategy vector from Theorem 1 is played. In turn, regardless of the marginal candidate's choice  $f_{m,d}^{P-*} \in \{0, \hat{f}_{m,d}^{P-}\}$ , it is straight-forward to show that (algebra omitted) that  $\{\mathbf{v}_d^{P-}(\mathbf{F}_d^*) + [(1 - 1_{S_d+2,d}^{P-}) / M_d]\} < \hat{\mathbf{v}}_d^{P-}(S_d + 2)$ . In words, even if the candidate at list

position  $(S_d + 2)$  deviated from  $\mathbf{F}_d^*$  in Theorem 1 so as to secure 100% voter support in her region,  $P_d^-$  would still receive no more than  $\mathbf{x}_d^{P-} = (S_d + 1)$  legislative seats, and the incumbent candidate at list position  $(S_d + 2)$  would not be re-elected.

\* Why is this the case? The marginal candidate never has the incentive to push  $P_d^-$ 's vote share higher than (C1), since this is all that is required for her re-election. As such, in order for the candidate at list position  $(S_d + 2)$  to secure her party an additional seat at  $\mathbf{F}_d^*$ , she would need to secure her party the equivalent of an entire additional electoral quota. Since we know that  $1_{S_d+2,d}^{P-} > 0$ , i.e. there is some non-zero number of party loyalists in the candidate at list position  $(S_d + 2)$ 's region, we also know that there will not be enough undecided voters in the candidate's region to secure an entire electoral quota. As such, by Lemma 2 this candidate will never deviate from  $\mathbf{F}_d^*$  in Theorem 1. This in turn implies that candidates from  $P_d^-$  at list positions  $(S_d + 3)$ ,  $(S_d + 4)$ , and so on cannot secure their own re-election, even if they receive 100% voter support in their respective regions. This demonstrates that no candidate below the marginal list position ever has the incentive to deviate from  $\mathbf{F}_d^*$ , thus establishing Existence.

## II. Theorem 1: Proof of Uniqueness

\* Consider a strategy vector  $\mathbf{F}_d$  in which some number  $K > 1$  incumbents from  $P_d^-$  choose  $f_{j,d}^{P-} > 0$ . At any such vector either: a.) all  $K$  gain re-election; or b.) at least one of the  $K$  does not gain re-election. If (b), then at least one incumbent has the incentive to defect by Lemma 2. If (a), then only the incumbent from among these  $K$  with the lowest list position might not have the incentive to defect. In contrast, all those incumbents from among the  $K$  with higher list positions would be able to decrease  $f_{j,d}^{P-}$  without losing re-election by free-riding on the regional vote share of the

incumbent from among these  $K$  with the lowest list position.

\* For example, consider a situation in which the candidates with list positions  $(S_d + 1)$  and  $(S_d + 2)$  both choose  $f_{j,d}^{P^-} > 0$  and gain re-election, such that  $\mathbf{x}_d^{P^-} = (S_d + 2)$ . In this case, the candidate with list position  $(S_d + 1)$  could reduce her regional effort to  $f_{m,d}^{P^-} = 0$  without losing re-election, since she only needs  $P_d^-$  to win  $\mathbf{x}_d^{P^-} = (S_d + 1)$  seats in order to be re-elected. The same is true anytime some number  $K > 1$  incumbents from  $P_d^-$  choose  $f_{j,d}^{P^-} > 0$  and all  $K$  gain re-election. As such, no strategy vector in which  $K > 1$  incumbents from  $P_d^-$  choose  $f_{j,d}^{P^-} > 0$  can be a NE.

\* It is straight-forward to show (algebra omitted) that, if  $K \leq 2$  incumbents from  $P_d^-$  choose  $f_{j,d}^{P^-} > 0$ , then all incumbents from  $P_d^+$  will be re-elected even if they choose  $f_{j,d}^{P^+} = 0$ . In turn, since we've established that no strategy vector  $\mathbf{F}_d$  in which  $K > 1$  incumbents from  $P_d^-$  choose  $f_{j,d}^{P^-} > 0$  can be a NE, it follows that no  $\mathbf{F}_d$  at which any incumbents from  $P_d^+$  choose  $f_{j,d}^{P^+} = 0$  will be a NE.

\* As such, in equilibrium at most one incumbent, an incumbent from  $P_d^-$ , ever chooses  $f_{j,d}^{P^-} > 0$ . This will never be an incumbent with a list position higher than  $(S_d + 1)$  since they receive safe seats when all incumbents from  $P_d^+$  choose  $f_{j,d}^{P^+} = 0$ . As well, this will never be an incumbent with a list position lower than  $(S_d + 1)$  since they would be choosing  $f_{j,d}^{P^-} > 0$  without gaining re-election (see the above proof of Existence), which is ruled out by Lemma 2. As a result,  $\mathbf{F}_d^*$  in Theorem 1 is the game's unique NE.

### III. NE when all Incumbents are Re-elected at the Full-Shirking Vector

\* Theorem 1 applies to districts in which party  $P_d^-$ 's incumbents are not all re-elected when the full-shirking vector  $\mathbf{F}_d^0$  is played. However, as noted at the outset, given **Assumption 1**, it is also possible that incumbents from both parties in a particular district are all re-elected when  $\mathbf{F}_d^0$  is played. In this case, it is straightforward to show that the full-shirking vector itself is the CLPR game's unique district-level NE (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^0$ ). The proof of Existence is trivial: since all incumbents secure re-election without devoting any effort to constituency service, choosing  $f_{j,d}^P > 0$  would represent a needless diversion of effort away from the pursuit of personal material gain. The proof of Uniqueness (available upon request) is largely identical to the Uniqueness proof presented in Sub-Section B above, demonstrating first that no number  $K > 1$  of either party's incumbents ever devote positive effort to constituency service; and in turn that, since at most 1 incumbent from either party ever devotes effort to constituency service, in equilibrium no incumbents devote any effort to constituency service.

## APPENDIX D. NASH EQUILIBRIA UNDER OLPR

\* This Appendix derives NE properties of the OLPR game laid out on page 25 of the text. In order to keep the Appendix to a reasonable length, the following material will not be included:

- a) Proofs of NE Uniqueness. The Appendix outlines Uniqueness proofs' methodology, but does not present them in full. All Uniqueness proofs are available upon request.
- b) Proofs of NE results in districts where  $\bar{A}_d = \bar{B}_d$ , i.e. in which both parties have the same number of current incumbents. The below results all pertain to districts in which one party has more current incumbents than the other. Strategically equivalent results for districts where  $\bar{A}_d = \bar{B}_d$  are also available upon request.

\* As noted in the text, this Appendix assumes that  $1_d$  is identical in all of district

Assumption is purely expository, and serves to mitigate the open-set problem that arises when actor's have continuous action sets. The game can be generalized to a situation in which incumbent legislators action sets are made up of infinitesimal but finite effort increments, in which case **Assumption 4** would be unnecessary. Candidate vote ties between incumbent candidates will be decided randomly and without bias.

### I. OLPR in Districts with High Levels of Party Loyalty

\* For a district  $d$  in which one party has more current incumbents than the other, define  $\bar{P}_d^{MA}(\bar{P}_d^{MI})$  as the number of seats currently held by the district's majority (minority) party (i.e.  $\bar{P}_d^{MA} > \bar{P}_d^{MI}$ ).

\* Proof of Existence: When  $\mathbf{F}_d^o$  is played the majority party receives aggregate district vote share:

$$\mathbf{v}_d^{MA}(\mathbf{F}_d^o) = \left\{ \frac{(\bar{P}_d^{MA} \cdot 1_d) + [\bar{P}_d^{MI} \cdot (1 - 1_d)]}{M_d} \right\}. \quad (\mathbf{D1})$$

\* **PROPOSITION 2a**: For districts in which  $\bar{P}_d^{MA} > \bar{P}_d^{MI}$ , if  $1_d > \left\{ 1 - \frac{1}{2 \cdot (2\bar{P}_d^{MA} - M_d)} \right\}$  then the full-shirking vector  $\mathbf{F}_d^o = \mathbf{F}_d^*$  is the OLPR game's unique district-level NE.

$d$ 's regions (**Assumption 2**), and that  $\beta_d, E^P \geq 1$  for all of district  $d$ 's incumbent legislators (**Assumption 3**). In Supplemental Appendix S2, I generalize the game to situations in which  $1_{j,d}^P$  varies across regions, in which  $\beta_{j,d}^P < 1$  for some or all incumbents, and in which  $E^P < 1$  for one or both parties.

\* As well, throughout this Appendix I will assume that, if an incumbent candidate and a non-incumbent candidate receive identical candidate vote shares and only one of the two can win a seat, the seat will go to the incumbent candidate (**Assumption 4**). This

\* By the quota and remainder rule, the majority party needs the following aggregate district vote share to win  $\mathbf{x}_d^{MA} = \bar{P}_d^{MA}$  seats:

$$(\bar{P}_d^{MA} / M_d) - (\frac{1}{2} \cdot M_d) + \varepsilon \quad (\varepsilon \rightarrow 0).^{45} \quad (\mathbf{D2})$$

\* By setting **(D1)** and **(D2)** equal to one another and solving for  $1_d$  we obtain the expression in Proposition 1. As long as  $1_d$  surpasses this level, at  $\mathbf{F}_d^o$  the majority and

<sup>45</sup> The above discussion of infinitesimal effort increments and the open-set problem is germane to all of this Appendix's derivations.

minority parties win  $\mathbf{x}_d^{MA} = \bar{P}_d^{MA}$  and  $\mathbf{x}_d^{MI} = \bar{P}_d^{MI}$  seats respectively via the quota remainder rule (algebra omitted), and these seats are allocated to the parties' incumbent candidates rather than their challengers, since at this level of party loyalty incumbents have higher candidate vote percentages than challenger candidates (i.e.  $V_{j,d}^P(0) > \{1 - V_{j,d}^P(0)\}$ ). As such, no incumbent wishes to deviate (Existence).

\* The proof of Uniqueness demonstrates first that no vector  $\mathbf{F}_d$  at which either party wins more seats than it currently holds is a NE; and then that no vector  $\mathbf{F}_d$  other than  $\mathbf{F}_d^*$  from Proposition 2a at which both parties win back their current number of seats is a NE.

implying that the majority party would win  $\mathbf{x}_d^{MA} = \bar{P}_d^{MA}$  seats with probability  $\frac{1}{2}$  and  $\mathbf{x}_d^{MA} = (\bar{P}_d^{MA} - 1)$  seats with probability  $\frac{1}{2}$ . If the latter were to occur, each majority party incumbent would have identical candidate vote shares, and would thus gain re-election with probability  $\pi_{j,d}^{MA} = [(\bar{P}_d^{MA} - 1) / \bar{P}_d^{MA}]$ . Thus, the majority incumbent who chooses  $f_{j,d}^{MA*} = \varepsilon$  ( $\varepsilon \rightarrow 0$ ) would rather exert infinitesimal effort increment and receive a seat with certainty than jeopardize her chances at re-election (Existence).

\* The proof of Uniqueness demonstrates first that no vector  $\mathbf{F}_d$  at which either party wins more seats than it currently holds is a

\* **PROPOSITION 2b:** For districts in which  $\bar{P}_d^{MA} > \bar{P}_d^{MI}$ , if  $l_d = \left\{ 1 - \frac{1}{2 \cdot (2\bar{P}_d^{MA} - M_d)} \right\}$  then the in the OLPR game's unique district-level NE one majority party incumbent chooses  $f_{j,d}^{MA*} = \varepsilon$  ( $\varepsilon \rightarrow 0$ ) and all the remaining incumbents choose  $f_{j,d}^{P*} = 0$ .

\* Proof of Existence: If incumbents behave as stipulated in Proposition 2b, the majority and minority parties win  $\mathbf{x}_d^{MA} = \bar{P}_d^{MA}$  and  $\mathbf{x}_d^{MI} = \bar{P}_d^{MI}$  seats via the quota remainder rule (algebra omitted), and these seats are allocated to the parties' incumbent candidates rather than their challengers, since at this level of party loyalty incumbents have higher candidate vote percentages than challenger candidates (i.e.  $V_{j,d}^P(0) > \{1 - V_{j,d}^P(0)\}$ ). Trivially, no

incumbent who chooses  $f_{j,d}^{P*} = 0$  wishes to deviate, as they gain re-election without devoting any effort to constituency service.

\* If the majority party incumbent who chooses  $f_{j,d}^{MA*} = \varepsilon$  ( $\varepsilon \rightarrow 0$ ) were to drop her constituency effort level to  $f_{j,d}^{MA} = 0$ , then the minority and majority parties would have identical remainder levels after their full set of quotas were subtracted from their aggregate district vote shares  $\mathbf{v}_d^P(\mathbf{F}_d^0)$ ,

NE; and then that no vector  $\mathbf{F}_d$  other than  $\mathbf{F}_d^*$  from Proposition 2b at which both parties win back their current number of seats is a NE.

## II. OLPR in Districts with Intermediate Levels of Party Loyalty

\* For districts in which  $\bar{P}_d^{MA} > \bar{P}_d^{MI}$ , I now identify the OLPR game's NE when district-level party loyalty is in the following range:

$$1 - \left\{ \frac{\bar{P}_d^{MA} - 1}{M_d} \right\} \leq l_d < 1 - \left\{ \frac{1}{2 \cdot (2\bar{P}_d^{MA} - M_d)} \right\}. \quad (\text{D3})$$

\* Using the quota remainder rule it is straightforward to show that, when  $l_d$  is in this range and  $\mathbf{F}_d^0$  is played, the minority party will win some number  $\mathbf{x}_d^{MI} > \bar{P}_d^{MI}$  seats and the majority party will win some number  $\mathbf{x}_d^{MA} < \bar{P}_d^{MA}$  seats (algebra omitted).

As such, by definition not all majority party incumbents will be re-elected when  $\mathbf{F}_d^0$  is played. By subtracting (D1) from (D2) we obtain:

$$\hat{\mathbf{v}}_d^{\text{MA}}(\mathbf{F}_d^0) = \left\{ \frac{[(2\bar{P}_d^{\text{MA}} - M_d) \cdot (1 - l_d)] - \frac{1}{2}}{M_d} \right\} + \varepsilon \quad (\varepsilon \rightarrow 0) \quad (\text{D4})$$

\* Assuming all minority party incumbents continue to choose  $f_{j,d}^{\text{MI}} = 0$ , expression (D4) represents the additional vote share needed by the majority party to secure itself  $\mathbf{x}_d^{\text{MA}} = \bar{P}_d^{\text{MA}}$  via the quota-remainder rule.

Define  $\hat{f}_d^{\text{MA}}$  as the constituency effort level that majority party incumbents would choose if they each were to devote *identical* levels of effort to constituency service, such that their combined efforts were *just sufficient* to win them  $\mathbf{x}_d^{\text{MA}} = \bar{P}_d^{\text{MA}}$  seats.

Formally,  $\hat{f}_d^{\text{MA}}$  is thus the constituency effort level at which  $(\bar{P}_d^{\text{MA}} \cdot \hat{f}_d^{\text{MA}}) / M_d = \hat{\mathbf{v}}_d^{\text{MA}}(\mathbf{F}_d^0)$ . Solving this for  $\hat{f}_d^{\text{MA}}$  yields:

$$\hat{f}_d^{\text{MA}} = \left\{ \frac{[(2\bar{P}_d^{\text{MA}} - M_d) \cdot (1 - l_d)] - \frac{1}{2}}{\bar{P}_d^{\text{MA}}} \right\} \quad (\text{D5})$$

\* Thus, if all majority party incumbents choose  $\hat{f}_d^{\text{MA}}$  they *divide evenly* the costs, in terms of constituency effort, of just barely gaining  $\mathbf{x}_d^{\text{MA}} = \bar{P}_d^{\text{MA}}$  seats.

\* **PROPOSITION 3:** If  $l_d$  is in the range (D3), then in the OLPR game's unique district-level NE one majority party incumbent chooses  $f_{j,d}^{\text{MA}*} = \hat{f}_d^{\text{MA}} + \varepsilon$  ( $\varepsilon \rightarrow 0$ ), all remaining majority party incumbents choose  $f_{j,d}^{\text{MA}*} = \hat{f}_d^{\text{MA}}$ , and all minority party incumbents choose  $f_{j,d}^{\text{MI}*} = 0$ .

\* Proof of Existence: When incumbents behave as stipulated in Proposition 3, the majority and minority parties win  $\mathbf{x}_d^{\text{MA}} = \bar{P}_d^{\text{MA}}$  and  $\mathbf{x}_d^{\text{MI}} = \bar{P}_d^{\text{MI}}$  seats respectively via the quota remainder rule, and these seats are allocated to the parties' incumbent candidates rather than their non-incumbents, since incumbents have higher candidate vote percentages. Trivially, no minority party incumbent has the incentive to deviate, since they gain re-election without devoting any effort to constituency service. Furthermore, no majority party who chooses incumbent  $f_{j,d}^{\text{MA}*} = \hat{f}_d^{\text{MA}}$  has the incentive to deviate:<sup>46</sup>

- a) devoting  $f_{j,d}^{\text{MA}} > \hat{f}_d^{\text{MA}}$  effort to constituency service is unnecessary for re-election;
- b) when incumbents behave as stipulated in Proposition 3, the majority party just barely secures enough district-level votes to win  $\mathbf{x}_d^{\text{MA}} = \bar{P}_d^{\text{MA}}$  legislative seats. As such, by dropping their constituency effort  $f_{j,d}^{\text{MA}} < \hat{f}_d^{\text{MA}}$ , a majority party incumbent would either drop their party's district-level vote share into a remainder tie with that of the minority party, or drop their seat share to  $\mathbf{x}_d^{\text{MA}} = (\bar{P}_d^{\text{MA}} - 1)$ . Furthermore, it would drop their own candidate vote share below that of the incumbents who choose

<sup>46</sup> Once again, the fact that no majority party incumbent who chooses  $f_{j,d}^{\text{MA}*} = \hat{f}_d^{\text{MA}}$  requires the assumption that  $\varepsilon$  is effectively 0, i.e. that no majority party incumbent who chooses  $f_{j,d}^{\text{MA}*} = \hat{f}_d^{\text{MA}}$  could drop her constituency effort by a 'lower' increment than  $\varepsilon$ . As with other 'open-set' issues, it is straight-forward (but less theoretically parsimonious...) to eliminate this problem by assuming that incumbents' action spaces are not continuous, but in fact composed of discreet but infinitesimal effort increments.



$f_{j,d}^{MA*} = \hat{f}_d^{MA}$ , implying that the deviating incumbent would no longer gain re-election with certainty. Since  $\beta_{j,d}^P \geq 1$ , there is no level  $f_{j,d}^{MA} < \hat{f}_d^{MA}$  at which the increased utility from personal enrichment outweighs this opportunity cost.

\* Finally the majority party incumbent who chooses  $f_{j,d}^{MA*} = \hat{f}_d^{MA} + \varepsilon$  has no incentive to deviate:

- a) devoting  $f_{j,d}^{MA} > \hat{f}_d^{MA}$  effort to constituency service is unnecessary for re-election;
- b) when incumbents behave as stipulated in Proposition 3, the majority party just barely secures enough district-level votes to win  $\mathbf{x}_d^{MA} = \bar{P}_d^{MA}$  legislative seats. As such, by dropping their constituency effort  $f_{j,d}^{MA} < \hat{f}_d^{MA}$ , a majority party incumbent would either drop their party's district-level vote share into a remainder tie with that of the minority party, or drop their seat share to  $\mathbf{x}_d^{MA} = (\bar{P}_d^{MA} - 1)$ , implying that the deviating incumbent would no longer gain re-election with certainty. Since  $\beta_{j,d}^P \geq 1$ , there is no

level  $f_{j,d}^{MA} < \hat{f}_d^{MA}$  at which the increased utility from personal enrichment outweighs this opportunity cost (Existence).

\* The proof of Uniqueness demonstrates first that no vector  $\mathbf{F}_d$  at which either party wins more seats than it currently holds is a NE; and then that no vector  $\mathbf{F}_d$  other than  $\mathbf{F}_d^*$  from Proposition 3 at which both parties win back their current number of seats is a NE.

### III. OLPR in Districts with Low Levels of Party Loyalty

\* For districts in which  $\bar{P}_d^{MA} > \bar{P}_d^{MI}$ , I now identify the OLPR game's NE when district-level party loyalty is in the following range:

$$l_d < 1 - \left\{ \frac{\bar{P}_d^{MA} - 1/2}{M_d} \right\}. \quad (\mathbf{D6})$$

\* Up to this point, all NE to the OLPR game have been Unique. I now demonstrate that, when  $l_d$  is in the range **(D6)**, the district-level OLPR game will have more than one possible NE. Nonetheless, these NE are confined to a narrow range of incumbents' action spaces, i.e. the model continues to generate precise and useful predictions as to the level of effort incumbent legislators will devote to constituency service in OLPR systems.

#### PROPOSITION 4: Mutually-Assured-Reelection in OLPR Districts

\* For districts in which  $\bar{P}_d^{MA} > \bar{P}_d^{MI}$ , if  $l_d$  is in the range **(D6)** then any district-level strategy vector  $\mathbf{F}_d$  which satisfies the following two criteria must be a NE to the OLPR game, and any NE to the OLPR game must satisfy the following two criteria (i.e. the criteria are both Necessary and Sufficient for the Existence of NE):

- a) all majority party incumbents choose an identical level of constituency effort  $f_{j,d}^{MA*}$ , and this level of constituency effort is in the range  $(l_d - 1_d) < f_{j,d}^{MA*} \leq (\bar{l}_d - 1_d)$ ;
- b) all minority party incumbents choose an identical level of constituency effort  $f_{j,d}^{MI*}$ , and this level of constituency effort is equal to  $f_{j,d}^{MI*} = [1 - (2l_d + f_{j,d}^{MA*})]$ .

\* Proof of Sufficiency: When incumbents behave as stipulated in Proposition 4, the majority and minority parties win  $\mathbf{x}_d^{MA} = \bar{P}_d^{MA}$  and  $\mathbf{x}_d^{MI} = \bar{P}_d^{MI}$  seats respectively via the quota remainder rule. Furthermore, the criteria  $f_{j,d}^{MI*} = [1 - (1_d + f_{j,d}^{MA*})]$  implies that both majority party incumbents and minority party incumbents receive an identical number of candidate votes as their parties' respective non-incumbent candidates (algebra omitted). As a result, all incumbent candidates are re-elected.

\* To prove that criteria (a) and (b) are Sufficient for the existence of NE, I establish that no incumbent has the incentive to deviate from any vector  $\mathbf{F}_d$  at which these criteria are satisfied:

- a) devoting  $f_{j,d}^P > f_{j,d}^{P*}$  effort to constituency service is unnecessary for re-election;
- b) at any strategy vector  $\mathbf{F}_d$  which satisfies these criteria, were any incumbent to drop their constituency effort to a level lower than  $f_{j,d}^P < f_{j,d}^{P*}$ , they would drop their candidate vote share to just below that of their party's non-incumbent challengers, implying that the deviating incumbent would no longer gain re-election. Since  $\beta_{j,d}^P \geq 1$  then, there is no level of effort  $f_{j,d}^P < f_{j,d}^{P*}$  at which the increased utility from personal enrichment outweighs this opportunity cost (Existence).

\* To prove that criteria (a) and (b) are Necessary conditions for the existence of NE, I must establish that any strategy vector  $\mathbf{F}_d$  which does satisfy these criteria is not a NE. As with the above proofs of Uniqueness, this derivation is omitted for reasons of space but available upon request (see Kselman 2008a). The proof of Necessity first demonstrates that no vector  $\mathbf{F}_d$  at which either party wins more seats than it currently holds is a NE; and then that no vector  $\mathbf{F}_d$  at which the parties win back

their current number of seats, but which does not satisfy Proposition 4's criteria, is a NE.

## APPENDIX E. SIMULATION ANALYSES

\* This Appendix begins by presenting the exogenous restrictions used in generating the simulation results plotted in FIGURES 5 and 6 (Section I), then moves to a step-by-step elaboration of the simulation process itself (Section II), and finally demonstrates that the qualitative hypotheses uncovered in FIGURES 5 and 6 are generalizable to any similar simulation analysis (Section III).

### I. Exogenous Restrictions

\* All of these simulations employ the following assumptions: the country's Legislature has  $N = 200$  seats; levels of party loyalty are uniform across all of a country's  $N = 200$  seats regions (define this uniform level of loyalty as  $l \in (0,1)$ ); and both  $\beta_{j,d}^P \geq 1$  for all incumbents and  $E^P \geq 1$  for both parties. The size of the Legislature and the uniformity of party loyalty are purely technical and have no bearing on the following simulations' generality.

\* As demonstrated in Supplemental Appendix S2, once incumbents' re-election utilities and parties' effort capacities fall below a certain level, the district-level OLPR game's NE properties may (or may not...) change depending on a district's incumbency status quo. For most values of  $\beta_{j,d}^P$  and  $E^P$  the OLPR game still generates stable NE outcomes, and higher aggregate levels of constituency effort than their FPTP and CLPR counterparts. On the other hand, for unusually low values of  $\beta_{j,d}^P$ , the OLPR game may under certain circumstances have no NE.

\* That said, this absence of NE does not in fact violate the basic comparative static hypotheses presented here: at these extremely low values of  $\beta_{j,d}^P$ , neither FPTP nor CLPR systems generate any constituency service (i.e. at these extremely low re-election utilities  $T^*(FPTP) = T^*(CLPR) = 0$ ; see Supplemental Appendix S2). As such, at these very low values of  $\beta_{j,d}^P$ , the fact that OLPR competition does not generate a

stable outcome in fact makes it more constituency oriented than either FPTP or CLPR systems, which generate stable outcomes characterized by the categorical absence of constituency service. As such, like the assumption that  $N = 200$  and that  $l \in (0,1)$  is uniform across regions, the assumptions that  $\beta_{j,d}^P \geq 1$  and  $E^P \geq 1$  do not affect the following analyses' generality.

### II. Deriving Total Aggregate Constituency Effort

\* FIGURES 5 and 6 present the statistic  $T^*(I)$  for all three institutions  $I \in \{FPTP, CLPR, OLPR\}$  at all possible values of  $l \in (0,1)$ . In FPTP systems, the  $N = 200$  incumbents represent single-member districts and the calculation of  $T^*(FPTP)$  is straightforward. For any value of  $l > \frac{1}{2}$  all incumbent legislators will choose  $f_{1,d}^{P*} = 0$ , since by the definition of plurality rule they can do so and still win back their legislative seat. In turn, if  $l > \frac{1}{2}$  then  $T^*(FPTP) = 0$ . On the other hand, recalling the analysis of FPTP elections from the text, if  $l < \frac{1}{2}$  then individual incumbents will have to devote  $\hat{f}_{1,d}^P = \frac{1}{2} - l + \epsilon$  ( $\epsilon \rightarrow 0$ ) in order to gain re-election. Since by construction  $\beta_{j,d}^P \geq 1$ , incumbents will always choose to exert the effort necessary for re-election. As such, for values of  $l < \frac{1}{2}$  we know that  $T^*(FPTP) = 200 \cdot \hat{f}_{1,d}^P \cong 100 - 200 \cdot l$ . These facts yield the plots of  $T^*(FPTP)$  in FIGURES 5 and 6.

\* In PR systems, FIGURE 5 begins with the case in which a country's 200 seats are divided into  $D = 20$  distinct regions, each with a magnitude of  $M_d = 10$ , and characterized by the following arbitrarily chosen district-by-district incumbency breakdown:

**TABLE E1. Proportional Simulation 1**

3 districts in which party $A$ has $\bar{A}_d = 8$ seats and party $B$ has $\bar{B}_d = 2$ seats.
3 districts in which party $A$ has $\bar{A}_d = 7$ seats and party $B$ has $\bar{B}_d = 3$ seats.
3 districts in which party $A$ has $\bar{A}_d = 6$ seats and party $B$ has $\bar{B}_d = 4$ seats.
3 districts in which party $A$ has $\bar{A}_d = 5$ seats and party $B$ has $\bar{B}_d = 5$ seats.
3 districts in which party $A$ has $\bar{A}_d = 4$ seats and party $B$ has $\bar{B}_d = 6$ seats.
3 districts in which party $A$ has $\bar{A}_d = 3$ seats and party $B$ has $\bar{B}_d = 7$ seats.
2 districts in which party $A$ has $\bar{A}_d = 2$ seats and party $B$ has $\bar{B}_d = 8$ seats.

\* This district-by-district breakdown implies a slim 103-to-97 legislative majority for party  $A$ . The first thing to note about the calculation of  $T^*(CLPR)$  and  $T^*(OLPR)$  is that, in both CLPR and OLPR systems, district-level NE outcomes are qualitatively unaffected by the identity of the district-level majority party. For example, the NE outcomes in districts where party  $A$  has  $\bar{A}_d = 6$  seats and party  $B$  has  $\bar{B}_d = 4$  seats are qualitatively identical to those of districts in which party  $A$  has  $\bar{A}_d = 4$  seats and party  $B$  has  $\bar{B}_d = 6$  seats; the NE outcomes in districts where party  $A$  has  $\bar{A}_d = 7$  seats and party  $B$  has  $\bar{B}_d = 3$  seats are qualitatively identical to those of districts in which party  $A$  has  $\bar{A}_d = 3$  seats and party  $B$  has  $\bar{B}_d = 7$  seats; and so on.

\* To demonstrate this point, begin with the derivation of  $T^*(CLPR)$ , and consider a district in which party  $A$  has  $\bar{A}_d = 6$  seats and party  $B$  has  $\bar{B}_d = 4$  seats, and in which regional loyalty is  $l = \frac{1}{5}$ . In this case, in equilibrium the marginal incumbent is the incumbent candidate from party  $A$  at list position 5: when the full-shirking vector is played,  $A$  receives a district-level vote share of  $v_d^A(\mathbf{F}_d^0) = [(\frac{1}{5} \cdot 6 + \frac{4}{5} \cdot 4)/10] = .44$ , which in turn implies that at the full-shirking vector party  $A$  wins  $x_d^A(\mathbf{F}_d^0) = 4$  seats. In order to secure his or her re-election, the marginal candidate must thus devote  $\hat{f}_{j,d}^A = \frac{1}{10} + \epsilon$  ( $\epsilon \rightarrow 0$ ), so as to push

party  $A$ 's remainder just above that received by party  $B$ . Since  $\beta_{j,d}^P \geq 1$  by construction, in the unique district-level NE the marginal candidate will choose  $f_{j,d}^{A*} = \hat{f}_{j,d}^A$  and all other incumbents will choose  $f_{j,d}^{P*} = 0$ .

\* Now move to a situation in which party  $A$  has  $\bar{A}_d = 4$  seats and party  $B$  has  $\bar{B}_d = 6$  seats, and in which regional loyalty is  $l = \frac{1}{5}$ . In this situation the marginal candidate will be the candidate from party  $B$  at list position 5, and this candidate will face the same choice just described when the marginal candidate was from party  $A$  (i.e.  $\hat{f}_{m,d}^B = \frac{1}{10} + \epsilon$ , where  $\epsilon \rightarrow 0$ ). In the unique district level NE the marginal candidate will choose  $f_{j,d}^{B*} = \hat{f}_{j,d}^B$  and all other incumbents will choose  $f_{j,d}^{P*} = 0$ .

\* As such, regardless of district level majority party's identity, the total amount of constituency effort generated across all incumbents in a CLPR district with a 6-to-4 incumbency status quo when  $l = \frac{1}{5}$  will be  $(\frac{1}{10} + \epsilon)$ , since this is the marginal incumbent's choice and all other incumbents choose  $f_{j,d}^{P*} = 0$ . For any particular level of  $l \in (0,1)$ , define  $T_{6/4}(CLPR, l)$  as the total constituency effort generated in a CLPR district with a 6-to-4 incumbency status quo, such that for example  $T_{6/4}(CLPR, \frac{1}{5}) = (\frac{1}{10} + \epsilon)$ . Similarly, for any level of  $l \in (0,1)$ , define  $T_{7/3}(CLPR, l)$  as the total constituency

effort generated in a district with a 7-to-3 incumbency status quo;  $T_{8/2}(CLPR,1)$  as the total constituency effort generated in a district with a 8-to-2 incumbency status quo; and so on.

\* The district-by-district breakdown in Table E1 implies a total of 5 districts with an 8-to-2 incumbency status quo, 6 districts with a 7-to-3 incumbency status quo, 6 districts with a 6-to-4 incumbency status quo, and 3 districts with a 5-to-5 incumbency status quo. In turn, for any particular value of  $l \in (0,1)$ , in order to generate the statistic  $T^*(CLPR)$  plotted in FIGURE 5 we must first solve for the district-level effort under all posited incumbency situations, and then aggregate these across the entire Legislature as follows:

$$T^*(CLPR) = 5 \cdot [T_{8/2}(CLPR,1)] + 6 \cdot [T_{7/3}(CLPR,1)] + 6 \cdot [T_{6/4}(CLPR,1)] + 3 \cdot [T_{5/5}(CLPR,1)] \quad (E1)$$

\* FIGURE 5 presents the statistic  $T^*(CLPR)$  defined in (E1) for all possible values of  $l \in (0,1)$ . The specific calculations of district-level effort under all posited incumbency situations, and for any particular value of  $l \in (0,1)$ , are available upon request.

\* Now move to the calculation of  $T^*(OLPR)$ . As in the CLPR case, the identity of the district-level majority party has no qualitative consequence for the district-level NE to the OLPR game. For example, referring back to Proposition 4 in Appendix D, consider a district in which party A has  $\bar{A}_d = 6$  seats and party B has  $\bar{B}_d = 4$  seats, and in which regional loyalty is  $l = \frac{1}{5}$ . In this case, given that by construction  $\beta_{j,d}^P \geq 1$  and  $E^P \geq 1$ , in any NE all of party A's incumbents choose  $f_{j,d}^{A*} \in [\frac{7}{20}, \frac{9}{20}]$  and all of party B's incumbents choose  $f_{j,d}^{B*} = [1 - (\frac{2}{5} + f_{j,d}^{A*})]$ . Similarly, in a district where party A has  $\bar{A}_d = 4$  seats and party B has  $\bar{B}_d = 6$  seats,

and in which regional loyalty is  $l = \frac{1}{5}$ , in any NE all of B's incumbents choose  $f_{j,d}^{B*} \in [\frac{7}{20}, \frac{9}{20}]$  and all of party A's incumbents choose  $f_{j,d}^{A*} = [1 - (\frac{2}{5} + f_{j,d}^{B*})]$ .

\* As is clear from this example and Proposition 4 in Appendix D, at lower values of partisanship the OLPR game yields a well-defined range of NE outcomes rather than a unique NE. Without loss of generality, in calculating  $T^*(OLPR)$  I will employ the NE outcome which yields the mean level of constituency service for the relevant value of  $l \in (0,1)$  and the extant incumbency status quo. For example, in the case described above (a 6-to-4 incumbency status quo and regional loyalty level of  $l = \frac{1}{5}$ ), in calculating  $T^*(OLPR)$  I will adopt the NE outcome in which incumbents from the district-level majority party choose  $f_{j,d}^{MA*} = \frac{2}{5}$  and incumbents from the district-level minority party choose  $f_{j,d}^{MI*} = \frac{1}{5}$ .

\* For any particular level of  $l \in (0,1)$ , define  $T_{6/4}(OLPR,1)$  as the total constituency effort generated in the 'mean' NE to the OLPR game for a district with a 6-to-4 incumbency status quo, such that for example

$$T_{6/4}(OLPR, \frac{1}{5}) = [(\hat{f}_{j,d}^{MA*} \cdot 6) + (\hat{f}_{j,d}^{MI*} \cdot 4)] = [\frac{2}{5} \cdot 6 + \frac{1}{5} \cdot 4] = 3.2.$$

Similarly, for any level of  $l \in (0,1)$ , define  $T_{7/3}(OLPR,1)$  as the total constituency effort generated in the 'mean' NE to the OLPR game for a district with a 7-to-3 incumbency status quo;  $T_{8/2}(OLPR,1)$  as the total constituency effort generated in the 'mean' NE to the OLPR game for a district with a 8-to-2 incumbency status quo; and so on. In turn, given the district-by-district incumbency breakdown presented in Table E1, the following formula expresses  $T^*(OLPR)$  for any level of  $l \in (0,1)$ :

$$T^*(OLPR) = 5 \cdot [T_{8/2}(OLPR,1)] + 6 \cdot [T_{7/3}(OLPR,1)] + 6 \cdot [T_{6/4}(OLPR,1)] + 3 \cdot [T_{5/5}(OLPR,1)] \quad (E2)$$

\* Figure 5 presents the statistic  $T^*(OLPR)$  defined in (E2) for all possible values of  $l \in (0,1)$ . The specific calculations of district-level effort under all posited incumbency situations, and for any particular value of  $l \in (0,1)$ , are available upon request.

\* Figure 6 undertakes an identical process to that described with respect to Figure 5, except that PR systems are now divided into  $D=80$  distinct regions, 40 of which have a magnitude of  $M_d=3$  and 40 of which have a magnitude of  $M_d=2$ , characterized by the following arbitrarily chosen district-by-district incumbency breakdown:

**TABLE E2. Proportional Simulation 2**

10 districts in which party A has $\bar{A}_d = 3$ seats and party B has $\bar{B}_d = 0$ seats.
10 districts in which party A has $\bar{A}_d = 2$ seats and party B has $\bar{B}_d = 1$ seats.
10 districts in which party A has $\bar{A}_d = 1$ seats and party B has $\bar{B}_d = 2$ seats.
10 districts in which party A has $\bar{A}_d = 0$ seats and party B has $\bar{B}_d = 3$ seats.
15 districts in which party A has $\bar{A}_d = 2$ seats and party B has $\bar{B}_d = 0$ seats.
15 districts in which party A has $\bar{A}_d = 1$ seats and party B has $\bar{B}_d = 1$ seats.
10 districts in which party A has $\bar{A}_d = 0$ seats and party B has $\bar{B}_d = 2$ seats.

\* This district-by-district breakdown implies a slim 105-to-95 legislative majority for party A. The FPTP simulation is identical to that presented in Figure 5. As for the PR simulation, district-level NE outcomes are once again qualitatively unaffected by the identity of the district-level majority party. As such, to derive the plots captured in Figure 5 we simply re-stipulate expressions (E1) and (E2) for this altered district-by-district incumbency breakdown. Figure 6 presents three plots for all possible values of  $l \in (0,1)$ . The specific calculations of district-level effort under all incumbency situations, and for any particular value of  $l \in (0,1)$ , are available upon request.

### III. The Generality of Institutional Hypotheses

\* The plots presented in Figures 5 and 6 paint a consistent and telling picture: at unusually high levels of partisanship all three systems generate little to no constituency service; and as partisanship begins to drop, OLPR quickly begins to outperform its counterparts in generating

constituency service, whereas the relative performance of CLPR and FPTP systems depends on both a country's partisanship and its district magnitude.

\* These comparative static hypotheses do not emerge as a result of the specific, arbitrarily chosen simulations described above, i.e. these results emerge regardless of the size of the Legislature and the district-by-district incumbency status quo. To demonstrate this, note first that that, without aggregating across the entire Legislature, in any particular district-to-district comparison from the above simulations OLPR outperforms its counterparts. For example, given a situation in which PR districts are of magnitude  $M_d=10$ , were we to compare  $T_{6/4}(OLPR,1)$  with  $T_{6/4}(CLPR,1)$ , and then compare both of these to the constituency effort generated in 10 individual FPTP districts of size  $M_d=1$ , we would obtain identical comparative statics to those uncovered in Figures 5 and 6: at all but unusually high levels of partisanship OLPR outperforms its counterparts in generating constituency service, while the relationship

between CLPR and FPTP systems depends on the level of partisanship.

\* The same pattern emerges were we to compare  $T_{7/3}(OLPR,1)$  with  $T_{7/3}(CLPR,1)$ , and then compare both of those to the constituency effort generated in 10 individual FPTP districts; and so on for any situation in which the size of PR districts is  $M_d = 10$ . As well, the same applies to the performance of OLPR vis a vis its counterparts in districts of magnitude  $M_d = 3$  or  $M_d = 2$ : regardless of the district-level incumbency breakdown, at all but unusually high levels of partisanship, OLPR generates greater district-level constituency service than its counterparts.

\* Put otherwise, the aggregate plots pictured in Figures 5 and 6 simply reproduce comparative static relationships which exist in all possible district-by-district comparisons. As such, we could have simulated any district-by-district incumbency status quo and obtained the same results. Furthermore, Kselman (2008a) shows that this generality applies not only to districts of magnitude  $M_d = 10$ ,  $M_d = 3$ , or  $M_d = 2$ , but to districts of any magnitude. Simply stated, at all but unusually high levels of partisanship, OLPR generates greater district-level constituency service than its CLPR and FPTP counterparts. As such, when aggregated across an entire Legislature, the same comparative statics will emerge regardless of the simulation in question.

\* That said, while the choice of simulation environment does not affect the model's qualitative hypotheses, it does affect their quantitative size. The district-by-district analysis demonstrates that OLPR has a particularly strong impact on constituency service incentives in districts tilted heavily towards one party or another, i.e. where the majority party's current seats far outweigh those of the minority party. As such, for example, the effect of OLPR on constituency service incentives would be particularly strong in countries where parties' electoral support tends to be strongly concentrated in particular electoral districts. In terms of the simulation plotted in Figure 5, the effect of OLPR on constituency service incentives would be

slightly stronger if all PR districts were characterized by an 8-to-2 majority to minority party seat ratio, but would be slightly weaker if all PR districts were characterized by an 6-to-4 majority to minority seat ratio. That said, the basic comparative static hypothesis, that OLPR outperforms the other two systems at all but the highest partisanship levels, obtains in all exogenous environments.

## APPENDIX F. MULTI-PARTY COMPETITION AND ABSTENTION

\* I now extend the model developed in Theoretical Appendices B through E to situations in which more than two parties compete, and in which voters may abstain. Section I introduces the notational and technical changes necessary to extend the game to multi-party competition. Sections II-IV then solve the model for FPTP, CLPR, and OLPR systems respectively. Finally, Section V demonstrates that the basic comparative static implications developed in Appendix E carry over to an environment with more than two parties and voter abstention.

### I. Extending the Theoretical Framework

\* Define  $\Theta_d \in \{A, B, \dots, \Psi_d\}$  as the set of all political parties competing in a district  $d$ , where  $\Psi_d \in \{C, D, \dots, Z\}$  represents the last party contained inside the set  $\Theta_d$ . For any region  $j$ , define the regional incumbent's party as  $P$ , and the set of remaining parties as  $P_d^- \in \{A, B, \dots, O, Q, \dots, \Psi_d\}$ . In the two-party model, we were able to use the single parameter  $\sigma_{i,j}$  to capture voters' partisan attitude towards parties  $A$  and  $B$ . Similarly, without the option of abstaining, all voters whose reservation utility was not met by the party of their regional incumbent simply transferred their vote to the opposing party. To model multi-party situations, I alter the model in two ways:

- a) I assume that voters have a party-specific partisan attitude  $\sigma_{i,j}^\Theta$  for each of the parties in the set  $\Theta_d \in \{A, B, \dots, \Psi_d\}$ , such that in each region there is a party specific distribution of attitudes for each party  $\sigma_j^\Theta \sim [\underline{\sigma}_j^\Theta, \bar{\sigma}_j^\Theta]$ , governed by distributional assumptions identical to those made on page 15 of the text;
- b) I assume that, in addition to voting against the party of their regional incumbent if  $u_{i,j}^P < \eta$ , there exists a second reservation threshold  $\lambda$  which defines voters' choice to abstain or turnout. More

specifically, if there exists no party  $\Theta_d \in \{A, B, \dots, \Psi_d\}$  such that  $u_{i,j}^\Theta \geq \lambda$ , then voter  $i$  abstains (formally,  $i$  abstains if  $\forall \Theta_d \in \{A, B, \dots, \Psi_d\}, u_{i,j}^\Theta < \lambda$ ).<sup>47</sup>

\* Given some regional distribution of partisan attitudes for the regional incumbent's party  $\sigma_j^P \sim [\underline{\sigma}_j^P, \bar{\sigma}_j^P]$ , and some effort allocation  $f_{j,d}^P$  by this regional incumbent, we can use a procedure identical to that employed in Appendix B to determine  $V_{j,d}^P(f_{j,d}^P)$ , the regional vote share of this incumbent's party.<sup>48</sup>

\* For voters not sufficiently satisfied to choose the party of their regional incumbent, there are two possibilities: a.) if their partisan attitude towards all of the remaining parties is not sufficiently positive to merit turning out (i.e. if  $\forall P_d^- \in \{A, B, \dots, O, Q, \dots, \Psi_d\}, \sigma_{i,j}^{P_d^-} < \lambda$ ), then they will abstain; b.) if their partisan attitude towards at least one of the remaining parties is sufficient to merit turning out (i.e. if  $\exists P_d^- \in \{A, B, \dots, O, Q, \dots, \Psi_d\} : \sigma_{i,j}^{P_d^-} \geq \lambda$ ), then they will turn out and vote for the party which yields them the highest utility.

\* Given our distributional assumptions, there will continue to exist some non-zero number of loyalists  $l_{j,d}^P \in (0,1)$  in each region, i.e. voters whose partisan bias towards the party of the regional incumbent is strong enough that they choose this party even if  $f_{j,d}^P = 0$ . Furthermore, define  $\alpha_{j,d}^{P_d^-}$  as the percentage of regional voters who choose  $P_d^- \in \{A, B, \dots, O, Q, \dots, \Psi_d\}$  when the

<sup>47</sup> Theoretically this second reservation parameter could be either greater than or less than  $\eta$  (i.e.  $\lambda \geq \eta$  or  $\lambda \leq \eta$ ), although it makes more intuitive sense for  $\lambda \leq \eta$ : if a voter is sufficiently pleased to choose the regional incumbent's party over other parties, he or she is also sufficiently pleased to turnout and vote).

<sup>48</sup> The only change is that, if  $\lambda > \eta$ , then the relevant reservation utility for determining  $V_{j,d}^P(f_{j,d}^P)$  becomes  $\lambda$  instead of  $\eta$ .



regional incumbent chooses  $f_{j,d}^P = 0$ ; and define  $\alpha_{j,d}^0$  as the percentage of regional voters who abstain when the regional incumbent chooses  $f_{j,d}^P = 0$ .

## II. The FPTP Model with Multi-Party Competition and Abstention

\* I now extend the FPTP analysis presented on pages 18-19 of the text to a multi-party context. Note first of all that if the regional incumbent's party receives more votes than any competitor party even though he or she chooses  $f_{1,d}^P = 0$  (i.e. if  $\forall P_d \in \{A, B, \dots, O, Q, \dots, \Psi_d\}, l_{1,d}^P > \alpha_{1,d}^{P^*}$ ), then by the definition of plurality rule the regional incumbent can choose  $f_{1,d}^P = 0$  and still win the district's seat. Trivially, in these circumstances  $f_{1,d}^{P^*} = 0$ .

\* On the other hand, if there exists at least one party whose vote share outpaces that of the regional incumbent's party when  $f_{1,d}^P = 0$  (i.e. if  $\exists P_d \in \{A, B, \dots, \Psi_d\}: l_{j,d}^P < \alpha_{j,d}^{P^*}$ ), then the incumbent from region  $j$  will have to exert positive constituency effort in order to win. Define  $\hat{f}_{1,d}^P$  as the critical level of constituency effort this incumbent must exert in order to win back his or her seat. In the two-party case without abstention, defining this critical level of effort was straightforward. In the multi-party game with abstention, the critical level of effort  $\hat{f}_{1,d}^P$  will be affected by the covariances of the regional partisanship distributions  $\sigma_j^\Theta \sim [\underline{\sigma}_j^\Theta, \bar{\sigma}_j^\Theta]$ .

\* For example, consider a region  $j$  in which parties  $A$ ,  $B$ , and  $C$  compete, and whose current incumbent is affiliated with party  $A$ . Furthermore, assume that  $l_{1,d}^A = \frac{1}{4}$ ,  $\alpha_{1,d}^B = \frac{1}{4}$ ,  $\alpha_{1,d}^C = \frac{1}{3}$ , and  $\alpha_{1,d}^0 = \frac{1}{6}$ , i.e. that when the regional incumbent chooses  $f_{1,d}^A = 0$  party  $C$  is the plurality winner and  $\frac{1}{6}$  of the region's voters abstain. To determine the value of  $\hat{f}_{1,d}^P$  we need to understand the covariance of  $\sigma_j^A$  with  $\sigma_j^B$  and  $\sigma_j^C$  respectively. More particularly, as the district incumbent begins to devote

effort to  $f_{1,d}^A$  and secure additional votes, are these additional votes being taken from party  $B$ 's vote share  $\alpha_{1,d}^B = \frac{1}{4}$ , from party  $C$ 's votes share  $\alpha_{1,d}^C = \frac{1}{3}$ , or from the set of voters who had previously abstained  $\alpha_{1,d}^0 = \frac{1}{6}$ ?<sup>49</sup> Since  $C$  is the plurality winner when  $f_{1,d}^A = 0$ , the critical effort level  $\hat{f}_{1,d}^A$  will be lower to the extent that effort devoted to  $f_{1,d}^A$  has an especially strong reductive effect on  $C$ 's vote share  $\alpha_{1,d}^C = \frac{1}{3}$ .

\* Aside from this added complexity in calculating  $\hat{f}_{1,d}^P$ , the FPTP game with more than two parties and voter abstention mirrors perfectly that summarized by Proposition 1 in the text: if  $\hat{f}_{1,d}^P < \beta_{j,d}^P, E^P$  then the incumbent from region  $j$  chooses  $f_{1,d}^{P^*} = \hat{f}_{1,d}^D$ , but if either  $\hat{f}_{1,d}^P > \beta_{j,d}^P$  or  $\hat{f}_{1,d}^P > E^P$  then the incumbent from region  $j$  chooses  $f_{1,d}^{P^*} = 0$ . In other words, due to the lack of game theoretic incentives in the FPTP game, the inclusion of additional parties into the theoretical setup is of little strategic consequence.

## III. The CLPR Game with Multi-Party Competition and Abstention

\* I now move to the CLPR model with more than two parties and voter abstention. I will employ **Assumption 1** (that incumbents are placed above non-incumbent candidates on party lists) to study the multi-party game; like Theorem 1, the following result can be generalized to any exogenous list-formation mechanism.

<sup>49</sup> For example, if those voters who have less favorable partisan attitudes for  $A$  also have more favorable attitudes for  $B$  but less favorable attitudes for party  $C$ , then the primary consequence of devoting effort to  $f_{1,d}^A$  will be to increase  $A$ 's vote-share and decrease  $B$ 's vote share. On the other hand, if those voters who have less favorable partisan attitudes for  $A$  also have more favorable attitudes for  $C$  but less favorable attitudes for party  $B$ , then the primary consequence of devoting effort to  $f_{1,d}^A$  will be to increase  $A$ 's vote-share and decrease  $C$ 's vote share.

Outside of the fact that more than two parties compete and voters may abstain, the game thus proceeds identically to that presented on page 22 of the text. The following Theorem is thus a generalization of Theorem 1 to the multi-party context:

played. If either there are not a sufficient number of undecided voters (i.e. non-loyalists...) in the marginal incumbent's district to secure his or her party an additional legislative seat, or if this marginal incumbent lacks the re-election

\* **THEOREM 2:** Consider a CLPR district  $d$  in which some number  $Q$  political parties compete for office. If candidates' list positions are determined by **Assumption 1**, then in any NE at most  $Q - 1$  incumbent candidate ever devote positive effort to constituency service.

\* A more complete proof of Theorem 2 is available upon request (see Kselman 2008a). For reasons of space, here I provide an outline of the proof for a district  $d$  in which three parties compete (i.e. in which  $\Theta_d \in \{A, B, C\}$ ). When three parties compete and **Assumption 1** is employed to determine candidate list positions, there are three possible district-level re-election outcomes when the full-shirking vector  $\mathbf{F}_d^o$  is played:

- a) all incumbent candidates from parties  $A$ ,  $B$ , and  $C$  are re-elected;
- b) all incumbent candidates from two of three parties are re-elected, but some subset of incumbents from the third party is not re-elected;
- c) all incumbent candidates from one of three parties are re-elected, and some subset of incumbents from both of the remaining two parties are not re-elected.

**i.) NE when all District-Level Incumbents are Re-elected**

\* If (a), then the unique NE to the district-level CLPR game is the full-shirking vector (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^o$ ).

**ii.) NE when all Incumbents from Two of the Three Parties are Re-elected**

\* If (b), this implies that there is one district-level incumbent candidate who finds him- or herself at a marginal list position when the full-shirking vector  $\mathbf{F}_d^o$  is

incentive and/or the effort capacity to secure this additional seat, the district-level CLPR game's unique NE (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^o$ ).

\* If the district's single marginal incumbent has the necessary regional undecided voters, the necessary re-election incentive, and the necessary effort capacity, he or she will have the incentive to deviate from  $\mathbf{F}_d^o$  and secure her party an additional legislative seat. Unlike with the derivation of Theorem 1 above, in which this additional seat was always taken from one of the opposing party's non-incumbent candidates, this additional seat may now come at the expense of an opposing party incumbent or an opposing party non-incumbent. If the additional seat is taken from an opposing party non-incumbent, then in the CLPR game's unique district-level NE the marginal incumbent chooses  $f_{m,d}^{P*} = \hat{f}_{m,d}^P$  and all other incumbents choose  $f_{j,d}^{\Theta*} = 0$ . If the marginal incumbent's additional seat comes at the expense of an opposing party incumbent, then the CLRR game will have no NE in district  $d$ , as the marginal incumbent from and the opposing party incumbent from whom this seat is taken will cycle endlessly in competing for this single seat.

**iii.) NE when all Incumbents from One of the Three Parties are Re-elected**

\* If (c), this implies that there are two incumbent candidates from district  $d$  who find themselves at marginal list positions when the full-shirking vector  $\mathbf{F}_d^o$  is played. If there are not enough undecided voters in

either of the marginal incumbents' regions to secure their party the votes necessary to gain an additional legislative seat, then the unique district-level NE to the CLPR game with two marginal incumbents will be the full-shirking vector (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^0$ ). Similarly, if both of the marginal incumbents lack either the incentive and/or the effort to secure re-election, then the unique district-level NE with two marginal incumbents will be the full-shirking vector (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^0$ ).

\* If one party's marginal incumbent has the necessary regional undecided voters, the necessary re-election incentive, and the necessary effort capacity to secure an additional seat, but the other party's marginal incumbent is lacking in at least one of these areas, then the district-level CLPR game's NE profile mirrors the case in which the game had only one marginal incumbent: either the game has no NE, or in the game's unique NE the marginal incumbent with capacity and incentive to secure an additional seat chooses  $f_{m,d}^{P*} = \hat{f}_{m,d}^P$  while all other incumbents choose  $f_{j,d}^{0*} = 0$ .

\* If both parties' marginal incumbents have the necessary undecided regional voters, re-election incentive, and effort capacity to secure an additional seat, then there are three possible outcomes:

§ the CLPR game has no NE, because both marginal incumbents engage in infinite cycling with an opposing party incumbent candidate, or with one another, over legislative seats;

§ in the CLPR game's unique district-level NE one marginal incumbent chooses  $f_{m,d}^{P*} = \hat{f}_{m,d}^P$  and secures her party an additional seat, while all other incumbents choose  $f_{j,d}^{0*} = 0$ ;

§ in the CLPR game's unique district-level NE both marginal incumbents devote just enough effort to constituency service so as to secure re-election.

\* Thus, for the case in which three parties compete in district  $d$ , in any district-level

NE at most two incumbent devotes positive effort to constituency service; and the unique NE to the CLPR game is often the full-shirking vector itself. As presented in Theorem 2, this result can be extended to any multi-party situation: in a game with  $Q$  political parties, there may (but may not...) arise NE in which  $Q-1$  incumbents devote positive effort to constituency service. This also serves to demonstrate that, as the number of parties (and the number of potential marginal incumbents...) in the CLPR game increases, so do its prospects for constituency service.

#### IV. The OLPR Game with Multi-Party Competition and Abstention

\* The OLPR model with greater than two political parties and voter abstention is significantly more numerically complicated than the two-party model. As such, Kselman (2008a) provides a series of NE derivations for specific situations rather than a general NE proof of the multi-party OLPR game. For reasons of space, here I outline a series of these examples to exhibit the multi-party game's basic intuitions.

\* I will examine a district  $d$  of magnitude  $M_d = 10$  in which three political parties  $\Theta_d \in \{A, B, C\}$  compete for office, in which  $l_d$  is uniform across the entire district, and in which both  $\beta_d \geq 1$  and  $E^P \geq 1$  (as with the two-party OLPR game, all of these restrictions can be relaxed). As well, I will assume the incumbency status quo is  $\bar{A}_d = 4$ ,  $\bar{B}_d = 3$ , and  $\bar{C}_d = 3$ . Outside of the added theoretical complexity, the game unfolds identically to that described on page 25 of the text.

\* Begin with the case in which  $l_d = \frac{1}{2}$  in all of district  $d$ 's regions. We must also specify the relevant values of  $\alpha_{j,d}^{P\sim}$  and  $\alpha_{j,d}^0$  in each region. Consider first a situation in which  $\alpha_{j,d}^0 = \frac{1}{10}$  in all regions (i.e. 10% of voters in all regions abstain when the local incumbent chooses  $f_{j,d}^P = 0$ ), and in which  $\alpha_{j,d}^{P\sim} = \frac{1}{5}$  for all non-incumbent parties (i.e. all non-incumbent parties receive a vote share of 20% when the local incumbent chooses  $f_{j,d}^P = 0$ ).

\* If  $l_d = \frac{1}{2}$ ,  $\alpha_{j,d}^0 = \frac{1}{10}$ , and  $\alpha_{j,d}^{P\sim} = \frac{1}{5}$ , the unique district-level NE to the OLPR game is the full-shirking vector (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^0$ ). This is the strategic equivalent of Proposition 2a from Appendix D: at the full-shirking vector, all parties win back exactly as many seats as they currently hold by the quota-remainder rule, and these seats go to incumbent legislators, whose candidate vote shares are higher than those of their parties' respective non-incumbent candidates.

\* Now move to a situation in which once again  $l_d = \frac{1}{2}$  and  $\alpha_{j,d}^0 = \frac{1}{10}$ , but in which the vote share of non-incumbent parties  $\alpha_{j,d}^{P\sim}$  is no longer uniform across parties regions when the full-shirking vector  $\mathbf{F}_d^0$  is played. In particular assume that, when  $\mathbf{F}_d^0$  is played, in regions whose current incumbent is from party  $A$  parties  $B$  and  $C$  receive  $\alpha_{j,d}^B = \frac{3}{10}$  and  $\alpha_{j,d}^C = \frac{1}{10}$  respectively; in regions whose current incumbent is from party  $B$  parties  $A$  and  $C$  receive  $\alpha_{j,d}^A = \frac{1}{10}$  and  $\alpha_{j,d}^C = \frac{3}{10}$  respectively; and in regions whose current incumbent is from party  $C$  parties  $A$  and  $B$  receive  $\alpha_{j,d}^A = \frac{1}{10}$  and  $\alpha_{j,d}^B = \frac{3}{10}$  respectively. In this case  $\mathbf{F}_d^0$  will not be a NE, because by the quota-remainder party  $A$  no longer wins back all of its seats, and each of  $A$ 's incumbent candidates will have the incentive to defect from  $\mathbf{F}_d^0$  so as to secure one of the party's limited legislative seats.

\* The NE in this second scenario will depend on the co-variances of the regional partisanship distributions  $\sigma_j^\Theta$ . To simplify the analysis, I will make the following assumption regarding these co-variances: an increase in the vote share of the regional incumbent's party impacts equally the remaining parties' vote shares  $\alpha_{j,d}^{P\sim}$ , as well as the abstention rate  $\alpha_{j,d}^0$ .

\* For example, if an incumbent from party  $A$  devotes enough effort to  $f_{j,d}^A$  so as to increase her party's regional vote share by 3% (thus climbing from  $V_{j,d}^A(0) = l_d = \frac{1}{2}$  to

$V_{j,d}^A(0) = .53$ ), this results in a 1% drop in the vote shares of parties  $B$  and  $C$  as well as a 1% drop in the abstention rate (thus moving from  $V_{j,d}^B(\cdot) = \alpha_{j,d}^B = \frac{3}{10}$  to  $V_{j,d}^B(\cdot) = .29$ ; from  $V_{j,d}^C(\cdot) = \alpha_{j,d}^C = \frac{1}{10}$  to  $V_{j,d}^C(\cdot) = .09$ ; and from  $\alpha_{j,d}^0 = \frac{1}{10}$  to an abstention rate of .09). Though not necessary for the following results, this assumption greatly simplifies the analysis and presentation.

\* Given this assumption as to co-variances of the regional partisanship distributions  $\sigma_j^\Theta$  and the conditions described above ( $l_d = \frac{1}{2}$ ,  $\alpha_{j,d}^0 = \frac{1}{10}$ , and  $\alpha_{j,d}^{P\sim}$  varying across to parties and regions according to the above stipulations), in the OLPR game's unique district-level NE one of party  $A$ 's incumbents chooses  $f_{j,d}^{A*} = \frac{3}{16} + \varepsilon$  ( $\varepsilon \rightarrow 0$ ), the remaining incumbents from party  $A$  choose  $f_{j,d}^{A*} = \frac{3}{16}$ , and all of the remaining incumbents choose  $f_{j,d}^{B*} = f_{j,d}^{C*} = 0$ . This result is strategically equivalent to that of Proposition 3 from Appendix D: when incumbents behave as such, all parties win back exactly as many seats as they currently hold by the quota-remainder rule, and these seats go to incumbent legislators, whose candidate vote shares are higher than those of their parties' respective non-incumbent candidates. Furthermore, incumbent legislators from party  $A$  split evenly the cost of re-electing their entire legislative contingent.

\* Now move to a situation in which regional partisanship is  $l_d = \frac{1}{10}$ , and begin with the case in which  $\alpha_{j,d}^0 = \frac{1}{10}$  (10% of voters in all regions abstain when the local incumbent chooses  $f_{j,d}^P = 0$ ), and in which  $\alpha_{j,d}^{P\sim} = \frac{2}{5}$  (each non-incumbent party receives a vote share of 40% when the local incumbent chooses  $f_{j,d}^P = 0$ ). Given the above assumption regarding co-variances of the regional partisanship distributions  $\sigma_j^\Theta$  and the stipulated conditions when  $\mathbf{F}_d^0$  is played, in the OLPR game's unique district-level NE one of party  $A$ 's incumbents chooses  $f_{j,d}^{A*} = \frac{9}{40} + \varepsilon$

( $\varepsilon \rightarrow 0$ ) and all remaining incumbents choose  $f_{j,d}^{\Theta*} = \frac{9}{40}$ .

\* This result is strategically equivalent to Proposition 4 in Appendix D: when incumbents behave as such, all parties win back exactly as many seats as they currently hold by the quota-remainder rule, and these seats go to incumbent legislators, whose candidate vote shares are identical to those of their parties' non-incumbent candidates. Furthermore, the individual incumbent from party  $A$  who chooses  $f_{j,d}^{A*} = \frac{9}{40} + \varepsilon$  has no incentive to deviate to  $f_{j,d}^{A*} = \frac{9}{40}$ : by doing so he or she would move party  $A$ 's aggregate district vote share into a time with that of party's  $B$  and  $C$ , such that each of party  $A$ 's incumbents would only win a fourth seat probabilistically; this is strictly-dominated by choosing  $f_{j,d}^{A*} = \frac{9}{40} + \varepsilon$  and winning a seat with certainty.

\* Now move to a situation in which once again  $l_d = \frac{1}{10}$  and  $\alpha_{j,d}^0 = \frac{1}{10}$ , but in which the vote share of non-incumbent parties  $\alpha_{j,d}^{P*}$  is no longer uniform across parties regions when the full-shirking vector  $\mathbf{F}_d^0$  is played. In particular assume that, when  $\mathbf{F}_d^0$  is played, in regions whose current incumbent is from party  $A$  parties  $B$  and  $C$  receive  $\alpha_{j,d}^B = \frac{1}{2}$  and  $\alpha_{j,d}^C = \frac{3}{10}$  respectively; in regions whose current incumbent is from party  $B$  parties  $A$  and  $C$  receive  $\alpha_{j,d}^A = \frac{3}{10}$  and  $\alpha_{j,d}^C = \frac{1}{2}$  respectively; and in regions whose current incumbent is from party  $C$  parties  $A$  and  $B$  receive  $\alpha_{j,d}^A = \frac{3}{10}$  and  $\alpha_{j,d}^B = \frac{1}{2}$  respectively.

\* Given the above assumption regarding co-variances of the regional partisanship distributions  $\sigma_j^{\Theta}$  and the stipulated conditions when  $\mathbf{F}_d^0$  is played, in the OLPR game's unique district-level NE one of party  $A$ 's incumbents will choose  $f_{j,d}^{A*} = .388 + \varepsilon$  ( $\varepsilon \rightarrow 0$ ), all of remaining incumbents from  $A$  will  $f_{j,d}^{A*} = .388$ , and all remaining incumbents will choose  $f_{j,d}^{B*} = f_{j,d}^{C*} = \frac{3}{10}$ . This result contains

strategic elements from Propositions 3 and 4 in Appendix D: when incumbents behave as such, incumbent parties win back exactly as many seats as they currently hold by the quota-remainder rule, and the candidate vote shares of incumbents from parties all parties  $B$  ( $C$ ) are identical to those of their parties' non-incumbents from regions in which the current incumbent is from party  $C$  ( $B$ ). Furthermore, when incumbents from party  $A$  behave as stipulated their party's vote share is just sufficient to secure them 4 legislative seats, i.e. at this level  $A$ 's incumbents split evenly the cost of re-electing the entire legislative contingent.

## V. Institutional Comparative Statics in the Multi-Party Model

\* Section IV presented NE results of the multi-party OLPR game in districts with four distinct exogenous environments. In order to intuitively communicate the multi-party model's comparative static implications, I now examine all three electoral systems' performance in these four exogenous environments. More particularly, I will compare the aggregate district-level constituency effort generated in PR districts of magnitude  $M_d = 10$  (with incumbency breakdown  $\bar{A}_d = 4$ ,  $\bar{B}_d = 3$ , and  $\bar{C}_d = 3$ ) to the aggregate constituency effort generated in 10 individual FPTP districts of magnitude  $M_d = 1$ . Kselman (2008a) demonstrates in a more general manner that, across an entire Legislature, OLPR generates higher levels of aggregate constituency service in multi-party environments than either FPTP or CLPR systems, regardless of the exogenous circumstances.

\* Begin with the case in which  $l_d = \frac{1}{2}$ , in which  $\alpha_{j,d}^0 = \frac{1}{10}$  in all regions (i.e. 10% of voters in all regions abstain when the local incumbent chooses  $f_{j,d}^P = 0$ ), and in which  $\alpha_{j,d}^{P*} = \frac{1}{5}$  for all non-incumbent parties (i.e. all non-incumbent parties receive a vote share of 20% when the local incumbent chooses  $f_{j,d}^P = 0$ ). In this case, in both CLPR and OLPR systems the district-level game's unique NE is the full-shirking

vector (i.e.  $\mathbf{F}_d^* = \mathbf{F}_d^0$ ); and in individual FPTP districts  $f_{j,d}^{\Theta*} = 0$  for all ten incumbents. Recalling the notation from Appendix E, this implies that aggregate constituency effort in all three systems is zero (i.e. that

$$\forall I \in \{FPTP, CLPR, OLPR\}, T^*(I) = 0).$$

\* Now move to a situation in which once again  $l_d = \frac{1}{2}$  and  $\alpha_{j,d}^0 = \frac{1}{10}$ , but in which the vote share of non-incumbent parties  $\alpha_{j,d}^{P\sim}$  is no longer uniform across parties regions when the full-shirking vector  $\mathbf{F}_d^0$  is played. In particular assume that, when  $\mathbf{F}_d^0$  is played, in regions whose current incumbent is from party *A* parties *B* and *C* receive  $\alpha_{j,d}^B = \frac{3}{10}$  and  $\alpha_{j,d}^C = \frac{1}{10}$  respectively; in regions whose current incumbent is from party *B* parties *A* and *C* receive  $\alpha_{j,d}^A = \frac{1}{10}$  and  $\alpha_{j,d}^C = \frac{3}{10}$  respectively; and in regions whose current incumbent is from party *C* parties *A* and *B* receive  $\alpha_{j,d}^A = \frac{1}{10}$  and  $\alpha_{j,d}^B = \frac{3}{10}$  respectively. I will make the same assumption as above regarding these co-variances: an increase in the vote share of the regional incumbent's party impacts equally the remaining parties' vote shares  $\alpha_{j,d}^{P\sim}$ , as well as the abstention rate  $\alpha_{j,d}^0$ .

\* As demonstrated in Section IV of this Appendix, given this exogenous situation in the OLPR game's unique district-level NE one of party *A*'s incumbents chooses  $f_{j,d}^{A*} = \frac{3}{16} + \epsilon$  ( $\epsilon \rightarrow 0$ ), the remaining incumbents from party *A* choose  $f_{j,d}^{A*} = \frac{3}{16}$ , and all of the remaining incumbents choose  $f_{j,d}^{B*} = f_{j,d}^{C*} = 0$  (which implies that  $T^*(OLPR) \cong 4 \cdot \frac{3}{16} = \frac{3}{4}$ ). In contrast, in this exogenous situation both of the remaining systems generate no constituency service in the aggregate: the unique NE to the district-level CLPR game is  $\mathbf{F}_d^* = \mathbf{F}_d^0$ ; and in individual FPTP districts  $f_{j,d}^{\Theta*} = 0$  for all ten incumbents (i.e.  $T^*(OLPR) = T^*(FPTP) = 0$ ). As such, despite the fact that the district-wide loyalty rate remained identical to that in the

previous example ( $l_d = \frac{1}{2}$ ), the change in non-incumbent parties' regional success rates  $\alpha_{j,d}^{P\sim}$  pushed up aggregate constituency service in OLPR districts, but not in the other two systems.

\* Now move to a situation in which regional partisanship is  $l_d = \frac{1}{10}$ , in which  $\alpha_{j,d}^0 = \frac{1}{10}$  (10% of voters in all regions abstain when the local incumbent chooses  $f_{j,d}^P = 0$ ), and in which  $\alpha_{j,d}^{P\sim} = \frac{2}{5}$  (each non-incumbent party receives a vote share of 40% when the local incumbent chooses  $f_{j,d}^P = 0$ ). As we saw in Section IV of this Appendix, given the above assumption regarding co-variances of the regional partisanship distributions  $\sigma_j^\Theta$  and the stipulated conditions when  $\mathbf{F}_d^0$  is played, in the OLPR game's unique district-level NE one of party *A*'s incumbents chooses  $f_{j,d}^{A*} = \frac{9}{40} + \epsilon$  ( $\epsilon \rightarrow 0$ ) and all remaining incumbents choose  $f_{j,d}^{\Theta*} = \frac{9}{40}$  (which implies that  $T^*(OLPR) \cong 10 \cdot \frac{9}{40} = 2.25$ ).

\* Similarly, it is straightforward to show and in individual FPTP districts  $f_{j,d}^{\Theta*} = \frac{9}{40}$  for all ten incumbents (which implies that  $T^*(FPTP) = 10 \cdot \frac{9}{40} = 2.25$ ), and that in the CLPR game's unique NE the marginal incumbent from party *A* chooses  $f_{m,d}^{A*} = \frac{9}{40} + \epsilon$  while all other incumbents choose  $f_{j,d}^{\Theta*} = 0$  (which implies that  $T^*(CLPR) \cong \frac{9}{40} \cong .225$ ). As such, in this situation both OLPR and FPTP outperform CLPR systems, and the former two generate largely identical levels of aggregate constituency effort.

\* This equivalence of OLPR and FPTP at lower levels of party loyalty arises due to the strict restriction that  $\alpha_{j,d}^{P\sim} = \frac{2}{5}$  for all non-incumbent parties in all regions. Once  $\alpha_{j,d}^{P\sim}$  is allowed to vary even slightly across regions and parties, this equivalence quickly dissipates. To demonstrate this, now move to a situation in which once again  $l_d = \frac{1}{10}$  and  $\alpha_{j,d}^0 = \frac{1}{10}$ , but in which the vote share of non-incumbent parties  $\alpha_{j,d}^{P\sim}$  is no longer uniform across parties

regions when the full-shirking vector  $\mathbf{F}_d^o$  is played. In particular assume that, when  $\mathbf{F}_d^o$  is played, in regions whose current incumbent is from party  $A$  parties  $B$  and  $C$  receive  $\alpha_{j,d}^B = \frac{1}{2}$  and  $\alpha_{j,d}^C = \frac{3}{10}$  respectively; in regions whose incumbent is from party  $B$  parties  $A$  and  $C$  receive  $\alpha_{j,d}^A = \frac{3}{10}$  and  $\alpha_{j,d}^C = \frac{1}{2}$  respectively; and in regions whose incumbent is from party  $C$  parties  $A$  and  $B$  receive  $\alpha_{j,d}^A = \frac{3}{10}$  and  $\alpha_{j,d}^B = \frac{1}{2}$  respectively.

\* As we saw in Section IV, given the above assumption regarding co-variances of the regional partisanship distributions  $\sigma_j^\Theta$  and the stipulated conditions when  $\mathbf{F}_d^o$  is played, in the OLPR game's unique district-level NE one of party  $A$ 's incumbents will choose  $f_{j,d}^{A*} = .388 + \epsilon$  ( $\epsilon \rightarrow 0$ ), all of remaining incumbents from  $A$  will  $f_{j,d}^{A*} = .388$ , and all remaining incumbents will choose  $f_{j,d}^{B*} = f_{j,d}^{C*} = \frac{3}{10}$  (which implies that  $T^*(OLPR) \cong 6 \cdot \frac{3}{10} + 4 \cdot .388 \cong 3.36$ ). Similarly, in this same situation it is straightforward to show and in individual FPTP districts  $f_{j,d}^{\Theta*} = \frac{3}{10}$  for all ten incumbents (which implies that  $T^*(FPTP) = 10 \cdot \frac{3}{10} = 3$ ), and that in the CLPR game's unique district-level NE is the full-shirking vector ( $\mathbf{F}_d^* = \mathbf{F}_d^o$ ). As such, OLPR outperforms FPTP, which itself outperforms CLPR systems, as occurred at lower levels of partisanship in the two-party game.

\* These results, which are generalized in Kselman (2008a), can be summarized as follows: at the highest levels of party loyalty, the systems will once again be largely indistinguishable, all generating little to no constituency effort. As levels of party loyalty drop, in all but the remotest circumstances OLPR will generate higher levels of constituency effort than its FPTP and CLPR counterparts. The only exception to this rule occurs when non-incumbent parties perform identically across all parties and regions, an unlikely scenario. Note that this Section's results apply only to a single district of magnitude  $M_d = 10$ . As such, the

conditions under which OLPR and FPTP generate identical levels of constituency effort are even more unlikely when examined at the level of an entire Legislature: across an entire Legislature, the two systems only generate identical levels of constituency effort when non-incumbent parties perform identically across all parties, in all regions, *in all districts*. Put simply, in all but the most unlikely cases, the basic comparative static hypotheses uncovered for the two-party game in Appendix E apply identically to the multi-party game.

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